

Este apunte fue puesto  
a disposición por el  
**Centro de Estudiantes**  
para la distribución digital



**Centro de Estudiantes  
de Ingeniería Tecnológica**

## NUMEROS NOTABLES

$$\pi = 3,1415\dots$$

$$\sqrt{2} = 1,414213\dots$$

$$e^\pi = 23,14069\dots$$

$$\sqrt{\pi} = 1,77245\dots = \Gamma\left(\frac{1}{2}\right) \quad (\Gamma : \text{función Gama})$$

$$\Gamma\left(\frac{1}{3}\right) = 2,67893\dots$$

$$\gamma = 0,57721566\dots \quad (\text{constante de Euler})$$

$$1 \text{ radián} = \frac{180^\circ}{\pi} = 57,29577\dots^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ radianes} = 0,01745\dots \text{ radianes}$$

$$\text{números de Euler } (E_x) = \frac{\pi^{2k+1}}{2^{2k+2} (2k)!} E_x$$

$$\text{números de Bernoulli } (B_x) = \frac{\pi^{2k} (2^{2k} - 1)}{2(2k)!} B_x$$

$$e = 2,718281\dots$$

$$\sqrt{3} = 1,73205\dots$$

$$\pi^e = 22,45915\dots$$

$$\sqrt{e} = 1,64872\dots$$

## FUNCIONES TRIGONOMETRICAS CIRCULARES

### RELACIONES ENTRE FUNCIONES TRIGONOMETRICAS

$$1) \quad \text{sen}^2 x + \text{cos}^2 x = 1$$

$$3) \quad \text{cotg } x = \frac{\text{cos } x}{\text{sen } x}$$

$$5) \quad \text{cosec } x = \frac{1}{\text{sen } x}$$

$$7) \quad 1 + \text{cotg}^2 x = \text{cosec}^2 x$$

$$2) \quad \text{tg } x = \frac{\text{sen } x}{\text{cos } x}$$

$$4) \quad \text{sec } x = \frac{1}{\text{cos } x}$$

$$6) \quad 1 + \text{tg}^2 x = \text{sec}^2 x$$

### FUNCIONES DE LA SUMA O DIFERENCIA DE ANGULOS

$$1) \quad \text{sen}(x \pm y) = \text{sen } x \text{ cos } y \pm \text{sen } y \text{ cos } x$$

$$2) \quad \text{cos}(x \pm y) = \text{cos } x \text{ cos } y \mp \text{sen } x \text{ sen } y$$

$$3) \quad \text{tg}(x \pm y) = \frac{\text{tg } x \pm \text{tg } y}{1 \mp \text{tg } x \text{ tg } y}$$

### FUNCIONES DEL DUPLO DEL ANGULO

$$1) \quad \text{sen } 2x = 2 \text{ sen } x \text{ cos } x$$

$$2) \quad \text{cos } 2x = \text{cos}^2 x - \text{sen}^2 x = 1 - 2 \text{ sen}^2 x = 2 \text{ cos}^2 x - 1$$

$$3) \quad \text{tg } 2x = \frac{2 \text{ tg } x}{1 - \text{tg}^2 x}$$

### FUNCIONES DEL ANGULO MITAD

$$1) \quad \text{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \text{cos } x}{2}}$$

$$2) \quad \text{cos}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \text{cos } x}{2}}$$

$$3) \quad \text{tg}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \text{cos } x}{1 + \text{cos } x}} = \frac{\text{sen } x}{1 + \text{cos } x} = \frac{1 - \text{cos } x}{\text{sen } x} = \text{cosec } x - \text{cotg } x$$

### FUNCIONES POTENCIA

$$1) \quad \text{sen}^2 x = \frac{1}{2} (1 - \text{cos } 2x)$$

$$2) \quad \text{cos}^2 x = \frac{1}{2} (1 + \text{cos } 2x)$$

## SUMA, DIFERENCIA Y PRODUCTO DE FUNCIONES

- 1)  $\operatorname{sen} x + \operatorname{sen} y = 2 \operatorname{sen}\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
- 2)  $\operatorname{sen} x - \operatorname{sen} y = 2 \cos\left(\frac{x+y}{2}\right) \operatorname{sen}\left(\frac{x-y}{2}\right)$
- 3)  $\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
- 4)  $\cos x - \cos y = -2 \operatorname{sen}\left(\frac{x+y}{2}\right) \operatorname{sen}\left(\frac{x-y}{2}\right)$
- 5)  $2 \operatorname{sen} x \operatorname{sen} y = \cos(x-y) - \cos(x+y)$
- 6)  $2 \cos x \cos y = \cos(x-y) + \cos(x+y)$
- 7)  $2 \operatorname{sen} x \operatorname{sen} y = \operatorname{sen}(x-y) + \operatorname{sen}(x+y)$

### FUNCIONES TRIGONOMETRICAS HIPERBOLICAS

- 1)  $\operatorname{senh} x = \frac{e^x - e^{-x}}{2}$
- 2)  $\operatorname{cosh} x = \frac{e^x + e^{-x}}{2}$
- 3)  $\operatorname{tgh} x = \frac{\operatorname{senh} x}{\operatorname{cosh} x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
- 4)  $\operatorname{cotgh} x = \frac{\operatorname{cosh} x}{\operatorname{senh} x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$
- 5)  $\operatorname{sech} x = \frac{1}{\operatorname{cosh} x}$
- 6)  $\operatorname{cosec} x = \frac{1}{\operatorname{senh} x}$

### RELACIONES FUNDAMENTALES

- 1)  $\operatorname{cosh}^2 x - \operatorname{senh}^2 x = 1$
- 2)  $\operatorname{tgh}^2 x + \operatorname{sech}^2 x = 1$
- 3)  $\operatorname{cotgh}^2 x - \operatorname{cosech}^2 x = 1$

### FUNCIONES DE LA SUMA Y DIFERENCIA DE ANGULOS

- 1)  $\operatorname{senh}(x \pm y) = \operatorname{senh} x \operatorname{cosh} y \pm \operatorname{senh} y \operatorname{cosh} x$
- 2)  $\operatorname{cosh}(x \pm y) = \operatorname{cosh} x \operatorname{cosh} y \pm \operatorname{senh} x \operatorname{senh} y$
- 3)  $\operatorname{tgh}(x \pm y) = \frac{\operatorname{tgh} x \pm \operatorname{tgh} y}{1 \pm \operatorname{tgh} x \operatorname{tgh} y}$

### FUNCIONES DEL DUPLO DEL ANGULO

- 1)  $\operatorname{senh} 2x = 2 \operatorname{senh} x \operatorname{cosh} x$
- 2)  $\operatorname{cosh} 2x = \operatorname{cosh}^2 x + \operatorname{senh}^2 x = 1 + 2 \operatorname{senh}^2 x = 2 \operatorname{cosh}^2 x - 1$
- 3)  $\operatorname{tgh} 2x = \frac{2 \operatorname{tgh} x}{1 - \operatorname{tgh}^2 x}$

### FUNCIONES DEL ANGULO MITAD

- 1)  $\operatorname{senh}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{\operatorname{cosh} x - 1}{2}} \begin{cases} \text{si } x > 0, \text{ vale signo } + \\ \text{si } x < 0, \text{ vale signo } - \end{cases}$
- 2)  $\operatorname{cosh}\left(\frac{x}{2}\right) = + \sqrt{\frac{1 + \operatorname{cosh} x}{2}}$
- 3)  $\operatorname{tgh}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{\operatorname{cosh} x - 1}{\operatorname{cosh} x + 1}} = \frac{\operatorname{senh} x}{1 + \operatorname{cosh} x} = \frac{\operatorname{cosh} x - 1}{\operatorname{senh} x}$

### RELACION ENTRE LOS ARGUMENTOS HIPERBOLICOS Y LOGARITMICOS

- 1)  $\operatorname{senh}^{-1} x = \ln\left(x + \sqrt{1 + x^2}\right); \quad \forall x$
- 2)  $\operatorname{tgh}^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right); \quad |x| < 1$
- 3)  $\operatorname{sech}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right); \quad 0 < x \leq 1$
- 4)  $\operatorname{cosh}^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right); \quad x \geq 1$
- 5)  $\operatorname{cotgh}^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{x-1}\right); \quad |x| > 1$

$$6) \operatorname{cosech}^{-1} x = \ln \left( \frac{1}{x} + \sqrt{\frac{1}{x^2} + 1} \right); \quad x \neq 0$$

### TABLA DE DERIVADAS

#### REGLAS DE DERIVACION

- Las funciones  $u$ ,  $v$  y  $w$  son derivables en  $x$ .
- $k$ ,  $r$ ,  $a$  y  $n$  son constantes reales.
- $x$  es variable independiente.

a) Regla de la cadena  $\frac{d}{dx} y = \frac{d}{du} y \cdot \frac{d}{dx} u$

b)  $\frac{d}{dx} y = \frac{1}{\frac{dy}{dx} x}$

c)  $\frac{d}{dx} y = \frac{\frac{d}{du} y}{\frac{d}{du} x}$

FUNCION	DERIVADA
$k$	$0$
$x$	$1$
$kx$	$k$
$ku$	$k \frac{d}{dx} u$
$u^r$	$r u^{r-1} \frac{d}{dx} u$
$u+v-w$	$\frac{d}{dx} u + \frac{d}{dx} v - \frac{d}{dx} w$
$uv$	$\frac{d}{dx} u \cdot v + u \cdot \frac{d}{dx} v$
$uvw$	$\frac{d}{dx} u \cdot v \cdot w + u \cdot \frac{d}{dx} v \cdot w + u \cdot v \cdot \frac{d}{dx} w$
$u/v$	$\frac{\frac{d}{dx} u \cdot v - u \cdot \frac{d}{dx} v}{v^2}; \quad v \neq 0$
$\ln x$	$1/x$
$\ln u$	$\frac{1}{u} \cdot \frac{d}{dx} u$
$\log_a u$	$\frac{1}{u \ln a} \frac{d}{dx} u \quad u > 0, a > 0, a \neq 1$
$e^x$	$e^x$
$a^u$	$a^u \ln a \frac{d}{dx} u$
$e^u$	$e^u \frac{d}{dx} u$
$u^v$	$u^v \left( \frac{d}{dx} v \ln u + \frac{v}{u} \frac{d}{dx} u \right); \quad u > 0$
$\operatorname{sen} u$	$\cos u \frac{d}{dx} u$
$\operatorname{cos} u$	$-\operatorname{sen} u \frac{d}{dx} u$
$\operatorname{tg} u$	$\sec^2 u \frac{d}{dx} u$
$\operatorname{cotg} u$	$-\operatorname{cosec}^2 u \frac{d}{dx} u$
$\operatorname{sec} u$	$\operatorname{sec} u \operatorname{tg} u \frac{d}{dx} u$
$\operatorname{cosec} u$	$-\operatorname{cosec} u \operatorname{cotg} u \frac{d}{dx} u$
$\operatorname{senh} u$	$\operatorname{cosh} u \frac{d}{dx} u$
$\operatorname{cosh} u$	$\operatorname{senh} u \frac{d}{dx} u$

$\operatorname{tgh} u$	$\operatorname{sech}^2 u \frac{d}{dx} u$
$\operatorname{cotgh} u$	$-\operatorname{cosec}^2 u \frac{d}{dx} u$
$\operatorname{sech} u$	$-\operatorname{sech} u \operatorname{tgh} u \frac{d}{dx} u$
$\operatorname{cosech} u$	$-\operatorname{cosech} u \operatorname{cotg} u \frac{d}{dx} u$
$\operatorname{sen}^{-1} u (\operatorname{arcsen} u)$	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$\operatorname{cos}^{-1} u (\operatorname{arccos} u)$	$-\frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$\operatorname{tg}^{-1} u (\operatorname{arctg} u)$	$\frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$\operatorname{cotg}^{-1} u (\operatorname{arccotg} u)$	$-\frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$\operatorname{sec}^{-1} u (\operatorname{arcsec} u)$	$\frac{1}{u\sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$\operatorname{cosec}^{-1} u (\operatorname{arccosec} u)$	$-\frac{1}{u\sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$\operatorname{senh}^1 u (\operatorname{arcsenh} u)$	$\frac{1}{\sqrt{u^2+1}} \frac{d}{dx} u$
$\operatorname{cosh}^{-1} u (\operatorname{arccosh} u)$	$\frac{1}{\sqrt{u^2-1}} \frac{d}{dx} u$
$\operatorname{tgh}^{-1} u (\operatorname{arctgh} u)$	$\frac{1}{1-u^2} \frac{d}{dx} u$
$\operatorname{cotgh}^{-1} u (\operatorname{arccotgh} u)$	$\frac{1}{1-u^2} \frac{d}{dx} u$
$\operatorname{sech}^{-1} u (\operatorname{arcsech} u)$	$-\frac{1}{u\sqrt{1-u^2}} \frac{d}{dx} u$
$\operatorname{cosech}^{-1} u (\operatorname{arccosech}^{-1} u)$	$-\frac{1}{u\sqrt{1+u^2}} \frac{d}{dx} u$

## TABLA DE INTEGRALES

### INTEGRALES INDEFINIDAS

#### REGLAS PARA UNA INTEGRACION

\* Las  $f, u, v$  y  $w$  son funciones de  $x$ .

\*  $a, b, q, r$  y  $n$  son constantes,  $r$  es real y  $n$  es natural.

1.  $\int a dx = ax$
2.  $\int a f(x) dx = a \int f(x) dx$
3.  $\int (u \pm v \pm w \pm \dots) dx = \int u dx \pm \int v dx \pm \int w dx \pm \dots$
4.  $\int u dv = uv - \int v du$  *Integración por partes*
5.  $\int f(ax) dx = \frac{1}{a} \int f(u) du$  *Cambio de variable  $u = ax$*
6.  $\int F\{f(x)\} dx = \int F(u) \frac{dx}{du} = \int \frac{F(u)}{f'(x)} du$
7.  $\int x^r dx = \frac{x^{r+1}}{r+1}$ ; Con  $r \neq -1$ . Para  $r = -1$  ver 8
8.  $\int \frac{1}{x} dx = \ln|x| = \begin{cases} \ln x & \text{si } x > 0 \\ \ln(-x) & \text{si } x < 0 \end{cases}$ ;  $x \neq 0$
9.  $\int e^x dx = e^x$

10.  $\int a^x dx = \frac{a^x}{\ln a} = a^x \log_a e$  Para  $a > 0$  y  $a \neq 1$
11.  $\int \operatorname{sen} x dx = -\cos x$
12.  $\int \cos x dx = \operatorname{sen} x$
13.  $\int \operatorname{tg} x dx = \ln \sec x = -\ln \cos x$
14.  $\int \operatorname{cotg} x dx = \ln \operatorname{sen} x$
15.  $\int \sec x dx = \ln(\sec x + \operatorname{tg} x) = \ln \operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{2}\right)$
16.  $\int \operatorname{cosec} x dx = \ln(\operatorname{cosec} x - \operatorname{cotg} x) = \ln \operatorname{tg} \frac{x}{2}$
17.  $\int \sec^2 x dx = \operatorname{tg} x$
18.  $\int \operatorname{cosec}^2 x dx = -\operatorname{cotg} x$
19.  $\int \operatorname{tg}^2 x dx = \operatorname{tg} x - x$
20.  $\int \operatorname{cotg}^2 x dx = -\operatorname{cotg} x - x$
21.  $\int \operatorname{sen}^2 x dx = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} = \frac{1}{2}(x - \operatorname{sen} x \cos x)$
22.  $\int \cos^2 x dx = \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} = \frac{1}{2}(x + \operatorname{sen} x \cos x)$
23.  $\int \sec x \operatorname{tg} x dx = \sec x$
24.  $\int \operatorname{cosec} x \operatorname{cotg} x dx = -\operatorname{cosec} x$
25.  $\int \operatorname{senh} x dx = \operatorname{cosh} x$
26.  $\int \operatorname{cosh} x dx = \operatorname{senh} x$
27.  $\int \operatorname{tgh} x dx = \ln \operatorname{cosh} x$
28.  $\int \operatorname{cotgh} x dx = \ln \operatorname{senh} x$
29.  $\int \operatorname{sech} x dx = \operatorname{sen}^{-1} x(\operatorname{tgh} x) \quad \acute{o} \quad 2 \operatorname{tg}^{-1} e^x$
30.  $\int \operatorname{cosech} x dx = \ln \operatorname{tgh} \frac{x}{2} \quad \acute{o} \quad -\operatorname{cot} \operatorname{gh}^{-1} e^x$
31.  $\int \operatorname{sech}^2 x dx = \operatorname{tgh} x$
32.  $\int \operatorname{cosech}^2 x dx = -\operatorname{cotgh} x$
33.  $\int \operatorname{tgh}^2 x dx = x - \operatorname{tgh} x$
34.  $\int \operatorname{cotgh}^2 x dx = x - \operatorname{cotgh} x$
35.  $\int \operatorname{senh}^2 x dx = \frac{\operatorname{senh} 2x}{4} - \frac{x}{2} = \frac{1}{2}(\operatorname{senh} x \operatorname{cosh} x - x)$
36.  $\int \operatorname{cosh}^2 x dx = \frac{\operatorname{senh} 2x}{4} + \frac{x}{2} = \frac{1}{2}(\operatorname{senh} x \operatorname{cosh} x + x)$
37.  $\int \operatorname{sech} x \operatorname{tgh} x dx = -\operatorname{sech} x$
38.  $\int \operatorname{cosech} x \operatorname{cotgh} x dx = -\operatorname{cosech} x$

39.  $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{tg}^{-1} \frac{x}{a}$
40.  $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left( \frac{x-a}{x+a} \right) = -\frac{1}{a} \operatorname{cotgh}^{-1} \frac{x}{a}; \quad x^2 > a^2$
41.  $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left( \frac{x+a}{a-x} \right) = \frac{1}{a} \operatorname{tgh}^{-1} \frac{x}{a}; \quad x^2 < a^2$
42.  $\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1} \frac{x}{a}$
43.  $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln(x + \sqrt{a^2+x^2}) \quad \text{ó} \quad \operatorname{senh}^{-1} \frac{x}{a}$
44.  $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x + \sqrt{x^2-a^2}) \quad \text{ó} \quad \operatorname{cosh}^{-1} \frac{x}{a}$
45.  $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \operatorname{sec}^{-1} \left| \frac{x}{a} \right|$
46.  $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a} \ln \left[ \frac{a+\sqrt{x^2+a^2}}{x} \right]$
47.  $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a} \ln \left[ \frac{a+\sqrt{a^2-x^2}}{x} \right]$
48.  $\int f^{(n)} g dx = f^{(n-1)} g - f^{(n-2)} g' + f^{(n-3)} g'' \dots + (-1)^n \int f \cdot g^{(n)} dx$

### METODO DE SUSTITUCION

49.  $\int F(ax+b) dx = \frac{1}{a} \int F(u) du \quad u = ax + b$
50.  $\int F(\sqrt{ax+b}) dx = \frac{2}{a} \int u F(u) du \quad u = \sqrt{ax+b}$
51.  $\int F(\sqrt[n]{ax+b}) dx = \frac{n}{a} \int u^{n-1} F(u) du \quad u = \sqrt[n]{ax+b}$
52.  $\int F(\sqrt{a^2-x^2}) dx = a \int F(a \cos u) \cos u du \quad x = a \operatorname{senu}$
53.  $\int F(\sqrt{x^2+a^2}) dx = a \int F(a \sec u) \sec^2 u du \quad x = a \operatorname{tgu}$
54.  $\int F(\sqrt{x^2-a^2}) dx = a \int F(a \sec u) \sec u \operatorname{tgu} du \quad x = a \operatorname{secu}$
55.  $\int F(e^{ax}) dx = \frac{1}{a} \int \frac{F(u)}{u} du \quad u = e^{ax}$
56.  $\int F(\ln u) dx = \int F(u) e^u du \quad u = \ln u$
57.  $\int F(\operatorname{sen}^{-1} \frac{x}{a}) dx = a \int F(u) \cos u du \quad u = \operatorname{sen}^{-1} \frac{x}{a}$

Para otras funciones trigonometricas reciprocas se obtienen similares resultados

58.  $\int F(\operatorname{sen} x \cdot \cos x) dx = 2 \int F\left(\frac{2u}{1+u^2} \cdot \frac{1-u^2}{1+u^2}\right) \frac{du}{1+u^2} \quad u = \operatorname{tg} \frac{x}{2}$

### Integrales indefinidas clasificadas por la forma

#### INTEGRALES CON $ax + b$

59.  $\int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b)$
60.  $\int \frac{x dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2} \ln(ax+b)$
61.  $\int \frac{x^2 dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3} \ln(ax+b)$
- 6 62.  $\int \frac{x^3 dx}{ax+b} = \frac{(ax+b)^2}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4} \ln(ax+b)$

63.  $\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln \left( \frac{x}{ax+b} \right)$
64.  $\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln \left( \frac{ax+b}{x} \right)$
65.  $\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3} \ln \left( \frac{x}{ax+b} \right)$
66.  $\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$
67.  $\int \frac{xdx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln(ax+b)$
68.  $\int \frac{x^2 dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3} \ln(ax+b)$
69.  $\int \frac{x^3 dx}{(ax+b)^2} = \frac{(ax+b)^2}{2a^4} - \frac{3b(ax+b)}{a^4} + \frac{b^3}{a^4(ax+b)} + \frac{3b^2}{a^4} \ln(ax+b)$
70.  $\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2} \ln \left( \frac{x}{ax+b} \right)$
71.  $\int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3} \ln \left( \frac{ax+b}{x} \right)$
72.  $\int \frac{dx}{x^3(ax+b)^2} = -\frac{(ax+b)^2}{2b^4x^2} - \frac{a^3x}{b^4(ax+b)} + \frac{3a(ax+b)}{b^4x} - \frac{3a^2}{b^4} \ln \left( \frac{ax+b}{x} \right)$
73.  $\int \frac{dx}{(ax+b)^3} = \frac{-1}{2(ax+b)^2}$
74.  $\int \frac{xdx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^2(ax+b)^2}$
75.  $\int \frac{x^2 dx}{(ax+b)^3} = \frac{2b}{a^3(ax+b)} - \frac{b^2}{2a^3(ax+b)^2} + \frac{1}{a^3} \ln(ax+b)$
76.  $\int \frac{x^3 dx}{(ax+b)^3} = \frac{x}{a^3} - \frac{3b^2}{a^4(ax+b)} + \frac{b^3}{2a^4(ax+b)} - \frac{3b}{a^4} \ln(ax+b)$
77.  $\int \frac{dx}{x(ax+b)^3} = \frac{a^2x^2}{2b^3(ax+b)^2} - \frac{2ax}{b^3(ax+b)} - \frac{1}{b^3} \ln \left( \frac{ax+b}{x} \right)$
78.  $\int \frac{dx}{x^2(ax+b)^3} = \frac{-a}{2b^2(ax+b)^2} - \frac{2a}{b^3(ax+b)} - \frac{1}{b^3x} + \frac{3a}{b^4} \ln \left( \frac{ax+b}{x} \right)$
79.  $\int \frac{dx}{x^3(ax+b)^3} = \frac{a^4x^3}{2b^5(ax+b)^2} - \frac{4a^3x}{b^5(ax+b)} - \frac{(ax+b)^2}{2b^5x^2} - \frac{6a^2}{b^5} \ln \left( \frac{ax+b}{x} \right)$
80.  $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a}$  Si  $n = -1$  véase 59
81.  $\int x(ax+b)^n dx = \frac{(ax+b)^{n+2}}{(n+2)a^2} - \frac{b(ax+b)^{n+1}}{(n+1)a^2}$ ;  $n \neq -1, -2$ .  
Si  $n = -1$  ó  $-2$  véase 62 ó 67, respectivamente.
82.  $\int x^2(ax+b)^n dx = \frac{(ax+b)^{n+3}}{(n+3)a^3} - \frac{b(ax+b)^{n+2}}{(n+2)a^3} + \frac{b^2(ax+b)^{n+1}}{(n+1)a^3}$ ;  $n \neq -1, -2, -3$   
Si  $n = -1, -2$  ó  $-3$  véase 61, 68 ó 75, respectivamente.

$$83. \int x^n (ax+b)^n dx = \begin{cases} \frac{x^{n+1}(ax+b)^n}{m+n+1} + \frac{nb}{m+n+1} \int x^n (ax+b)^{n-1} dx \\ \frac{x^m(ax+b)^{n+1}}{m+n+1} - \frac{mn}{(m+n+1)a} \int x^{m-1}(ax+b)^n dx \\ \frac{-x^{m+1}(ax+b)^{n+1}}{(n+1)b} + \frac{m+n+2}{(n+1)b} \int x^m(ax+b)^{n+1} dx \end{cases}$$

INTEGRALES CON  $\sqrt{ax+b}$

84.  $\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$
85.  $\int \frac{xdx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2} \sqrt{ax+b}$
86.  $\int \frac{x^2 dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2 - 4abx + 8b^2)}{15a^2} \sqrt{ax+b}$

$$87. \int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{b} \ln \left( \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} \right) & b \neq 0 \\ \frac{2}{\sqrt{-b}} \operatorname{tg}^{-1} \sqrt{\frac{ax+b}{-b}} & \end{cases}$$

$$88. \int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax+b}} ; \text{ Véase 87 } b \neq 0$$

$$89. \int \sqrt{ax+b} \cdot dx = \frac{2\sqrt{(ax+b)^3}}{3a}$$

$$90. \int x \sqrt{ax+b} \cdot dx = \frac{2(3ax-2b)}{15a^2} \sqrt{(ax+b)^3}$$

$$91. \int x^2 \sqrt{ax+b} \cdot dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3} \sqrt{(ax+b)^3}$$

$$92. \int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}} \quad \text{Véase 87}$$

$$93. \int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}} \quad \text{Véase 87}$$

$$94. \int \frac{x^m dx}{\sqrt{ax+b}} = \frac{2x^m \sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a} \int \frac{x^{m-1} dx}{\sqrt{ax+b}}$$

$$95. \int \frac{dx}{x^n \sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2m-3)a}{(2m-2)b} \int \frac{dx}{x^{m-1} \sqrt{ax+b}} ; m \neq 1$$

$$96. \int x^m \sqrt{ax+b} dx = \frac{2x^m}{(2m+3)a} \sqrt{(ax+b)^3} - \frac{2mb}{(2m+3)a} \int x^{m-1} \sqrt{ax+b} dx$$

$$97. \int \frac{\sqrt{ax+b}}{x^m} dx = -\frac{\sqrt{ax+b}}{(m+1)x^{m-1}} + \frac{a}{2(m-1)} \int \frac{dx}{x^{m-1} \sqrt{ax+b}}$$

$$98. \int \frac{\sqrt{ax+b}}{x^m} dx = -\frac{\sqrt{(ax+b)^3}}{(m-1)bx^{m-1}} - \frac{(2m-5)a}{(2m-2)b} \int \frac{\sqrt{ax+b}}{x^{m-1}} dx$$

$$99. \int \sqrt{(ax+b)^m} dx = \frac{2\sqrt{(ax+b)^{m+2}}}{a(m+2)}$$

$$100. \int x \sqrt{(ax+b)^m} dx = \frac{2\sqrt{(ax+b)^{m+4}}}{a^2(m+4)} - \frac{2b\sqrt{(ax+b)^{m+2}}}{a^2(m+2)}$$

$$101. \int x^2 \sqrt{(ax+b)^m} dx = \frac{2\sqrt{(ax+b)^{m+6}}}{a^3(m+6)} - \frac{4b\sqrt{(ax+b)^{m+4}}}{a^3(m+4)} + \frac{2b^2\sqrt{(ax+b)^{m+2}}}{a^3(m+2)}$$

$$102. \int \frac{\sqrt{(ax+b)^m}}{x} dx = 2 \frac{\sqrt{(ax+b)^m}}{m} + b \int \frac{\sqrt{(ax+b)^{m-2}}}{x} dx$$

$$103. \int \frac{\sqrt{(ax+b)^m}}{x^2} dx = -\frac{\sqrt{(ax+b)^{m+2}}}{bx} + \frac{ma}{2b} \int \frac{\sqrt{(ax+b)^m}}{x} dx$$

$$104. \int \frac{dx}{x \sqrt{(ax+b)^m}} = \frac{2}{(m-2)b \sqrt{(ax+b)^{m-2}}} + \frac{1}{b} \int \frac{dx}{x^m \sqrt{(ax+b)^{m-2}}}$$

**INTEGRALES CON  $ax+b$  y  $px+q$ , donde  $bp-aq \neq 0$**

$$105. \int \frac{1}{(ax+b)(px+q)} dx = \frac{1}{bp-aq} \ln \left( \frac{px+q}{ax+b} \right)$$

$$106. \int \frac{x}{(ax+b)(px+q)} dx = \frac{1}{bp-aq} \left( \frac{b}{a} \ln(ax+b) - \frac{q}{p} \ln(px+q) \right)$$

$$107. \int \frac{1}{(ax+b)^2(px+q)} dx = \frac{1}{bp-aq} \left( \frac{1}{ax+b} + \frac{p}{bp-aq} \ln \left[ \frac{px+q}{ax+b} \right] \right)$$

$$108. \int \frac{x}{(ax+b)^2(px+q)} dx = \frac{1}{bp-aq} \left( \frac{q}{bp-aq} \ln \left[ \frac{ax+b}{px+q} \right] - \frac{b}{a(ax+b)} \right)$$

$$109. \int \frac{x^2 dx}{(ax+b)^2(px+q)} = \frac{b^2}{(bp-aq)a^2(ax+b)} + \frac{1}{(bp-aq)^2} \left( \frac{q^2}{p} \ln(px+q) + \frac{b(bp-aq)}{a^2} \ln(ax+b) \right)$$

$$110. \int \frac{dx}{(ax+b)^m(px+q)^n} dx = \frac{-1}{(n-1)(bp-aq)} \left( \frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2) \int \frac{dx}{(ax+b)^m(px+q)^{n-1}} \right)$$

$$111. \int \frac{ax+b}{px+q} dx = \frac{ax}{p} + \frac{bp-aq}{p^2} \ln(px+q)$$

$$112. \int \frac{(ax+b)^m}{(px+q)^n} dx = \begin{cases} \frac{-1}{(n-1)(bp-aq)} \left( \frac{(ax+b)^{m+1}}{(px+q)^{n-1}} + a(n-m-2) \int \frac{(ax+b)^m}{(px+q)^{n-1}} dx \right) \\ \frac{-1}{(n-m-1)p} \left( \frac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq) \int \frac{(ax+b)^{m-1}}{(px+q)^n} dx \right) \\ \frac{-1}{(n-1)p} \left( \frac{(ax+b)^m}{(px+q)^{n-1}} - ma \int \frac{(ax+b)^{m-1}}{(px+q)^{n-1}} dx \right) \end{cases}$$

### INTEGRALES CON $\sqrt{ax+b}$ y $px+q$

$$113. \int \frac{px+q}{\sqrt{ax+b}} dx = \frac{2(apx+3aq-2bp)}{3a^2} \sqrt{ax+b}$$

$$114. \int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{bp-aq}\sqrt{p}} \ln \left( \frac{\sqrt{p(ax+b)} - \sqrt{bp-aq}}{\sqrt{p(ax+b)} + \sqrt{bp-aq}} \right) \\ \frac{2}{\sqrt{aq-bp}\sqrt{p}} \operatorname{tg}^{-1} \sqrt{\frac{p(ax+b)}{aq-bp}} \end{cases}$$

$$115. \int \frac{\sqrt{ax+b}}{px+q} dx = \begin{cases} \frac{2\sqrt{ax+b}}{p} + \frac{\sqrt{bp-aq}}{p\sqrt{q}} \ln \left( \frac{\sqrt{p(ax+b)} - \sqrt{bp-aq}}{\sqrt{p(ax+b)} + \sqrt{bp-aq}} \right) \\ \frac{2\sqrt{ax+b}}{p} - \frac{2\sqrt{aq-bp}}{p\sqrt{q}} \operatorname{tg}^{-1} \sqrt{\frac{p(ax+b)}{aq-bp}} \end{cases}$$

$$116. \int (px+q)^n \sqrt{ax+b} dx = \frac{2(px+q)^{n+1} \sqrt{ax+b}}{(2n+3)p} + \frac{bp-aq}{(2n+3)p} \int \frac{(px+q)^n}{\sqrt{ax+b}} dx$$

$$117. \int \frac{dx}{(px+q)^n \sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)(aq-bp)(px+q)^{n-1}} + \frac{(2n-3)a}{2(n-1)(aq-bp)} \int \frac{dx}{(px+q)^{n-1} \sqrt{ax+b}}$$

$$118. \int \frac{(px+q)^n}{\sqrt{ax+b}} = \frac{2(px+q)^n \sqrt{ax+b}}{(2n+1)a} + \frac{2n(aq-bp)}{(2n-1)p} \int \frac{(px+q)^{n-1}}{\sqrt{ax+b}} dx$$

$$119. \int \frac{\sqrt{ax+b}}{(px+q)^n} dx = \frac{-\sqrt{ax+b}}{(n-1)p(px+q)^{n-1}} + \frac{a}{2(n-1)p} \int \frac{dx}{(px+q)^{n-1} \sqrt{ax+b}}$$

### INTEGRALES CON $\sqrt{ax+b}$ y $\sqrt{px+q}$

$$120. \int \frac{dx}{(ax+b)(px+q)} = \begin{cases} \frac{2}{\sqrt{ap}} \ln \left( \sqrt{a(px+q)} + \sqrt{p(ax+b)} \right) \\ \frac{2}{\sqrt{-ap}} \operatorname{tg}^{-1} \sqrt{\frac{-p(ax+b)}{a(px+q)}} \end{cases}$$

$$121. \int \frac{xdx}{\sqrt{(ax+b)(px+q)}} = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap} \int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$122. \int \sqrt{(ax+b)(px+q)} dx = \frac{2apx+bp+aq}{4ap} \sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap} \int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$123. \int \sqrt{\frac{px+q}{ax+b}} dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{(aq-bp)}{2a} \int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

$$124. \int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$$

### INTEGRALES CON $x^2+a^2$

$$125. \int \frac{dx}{x^2+a^2} = \frac{1}{a} \operatorname{tg}^{-1} \frac{x}{a}$$

$$126. \int \frac{xdx}{x^2+a^2} = \frac{1}{2} \ln(x^2+a^2)$$

$$127. \int \frac{x^2 dx}{x^2+a^2} = x - a \operatorname{tg}^{-1} \frac{x}{a}$$

128.  $\int \frac{x^3 dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2} \ln(x^2 + a^2)$
129.  $\int \frac{dx}{x(x^2+a)} = \frac{1}{2a^2} \ln(x^2 + a^2)$
130.  $\int \frac{dx}{x^2(x^2+a)} = -\frac{1}{a^2 x} - \frac{1}{a^3} \operatorname{tg}^{-1} \frac{x}{a}$
131.  $\int \frac{dx}{x^3(x^2+a^2)} = -\frac{1}{2a^2 x^2} - \frac{1}{2a^4} \ln\left(\frac{x^2}{x^2+a^2}\right)$
132.  $\int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3} \operatorname{tg}^{-1} \frac{x}{a}$
133.  $\int \frac{x dx}{(x^2+a^2)} = \frac{-1}{2(x^2+a^2)}$
134.  $\int \frac{x^2 dx}{(x^2+a^2)^2} = \frac{-x}{2(x^2+a^2)} + \frac{1}{2a} \operatorname{tg}^{-1} \frac{x}{a}$
135.  $\int \frac{x^3 dx}{(x^2+a^2)^2} = \frac{a^2}{2(x^2+a^2)} + \frac{1}{2} \ln(x^2 + a^2)$
136.  $\int \frac{dx}{x(x^2+a^2)^2} = \frac{1}{2a^2(x^2+a^2)} + \frac{1}{2a^4} \ln\left(\frac{x^2}{x^2+a^2}\right)$
137.  $\int \frac{dx}{x^2(x^2+a^2)^2} = -\frac{1}{a^4 x} - \frac{x}{2a^4(x^2+a^2)} - \frac{3}{2a^5} \operatorname{tg}^{-1} \frac{x}{a}$
138.  $\int \frac{dx}{x^3(x^2+a^2)^2} = -\frac{1}{2a^4 x^2} - \frac{1}{2a^4(x^2+a^2)} - \frac{1}{a^6} \ln\left(\frac{x^2}{x^2+a^2}\right)$
139.  $\int \frac{dx}{(x^2+a)^n} = \frac{x}{2(n-1)a^2(x+a)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(x^2+a)^{n-1}}$ ; Si n=1 Ver 125
140.  $\int \frac{x dx}{(x^2+a^2)^n} = \frac{-1}{2(n-1)(x^2+a^2)^{n-1}}$  Si n=1 Ver 126
141.  $\int \frac{dx}{x(x^2+a^2)^n} = \frac{1}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{1}{a^2} \int \frac{dx}{x(x^2+a^2)^{n-1}}$ ; Si n=1 Ver 129
142.  $\int \frac{x^m dx}{(x^2+a^2)^n} = \int \frac{x^{m-2} dx}{(x^2+a^2)^{n-1}} - a^2 \int \frac{x^{m-2}}{(x^2+a^2)^n}$
143.  $\int \frac{dx}{x^m(x^2+a^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^m(x^2+a^2)^{n-1}} - \frac{1}{a^2} \int \frac{dx}{x^{m-2}(x^2+a^2)^n}$

INTEGRALES CON  $x^2 - a^2$ ;  $x^2 > a^2$

144.  $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right)$  ó  $\frac{-1}{a} \operatorname{cot} \operatorname{gh}^{-1} \frac{x}{a}$
145.  $\int \frac{x dx}{x^2-a^2} = \frac{1}{2} \ln(x^2 - a^2)$
146.  $\int \frac{x^2 dx}{x^2-a^2} = x + \frac{a}{2} \ln\left(\frac{x-a}{x+a}\right)$
147.  $\int \frac{x^3 dx}{x^2-a^2} = \frac{x^2}{2} + \frac{a^2}{2} \ln(x^2 - a^2)$
148.  $\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2} \ln\left(\frac{x^2-a^2}{x^2}\right)$
149.  $\int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2 x} + \frac{1}{2a^3} \ln\left(\frac{x-a}{x+a}\right)$
150.  $\int \frac{dx}{x^3(x^2-a^2)} = \frac{1}{2a^2 x^2} - \frac{1}{2a^4} \ln\left(\frac{x^2}{x^2-a^2}\right)$
151.  $\int \frac{dx}{(x^2-a^2)} = \frac{-x}{2a^2(x^2-a^2)} - \frac{1}{2a^4} \ln\left(\frac{x-a}{x+a}\right)$
152.  $\int \frac{x dx}{(x^2-a^2)^2} = \frac{-1}{2(x^2-a^2)}$
153.  $\int \frac{x^2 dx}{(x^2-a^2)^2} = \frac{-x}{2(x^2-a^2)} + \frac{1}{4a} \ln\left(\frac{x-a}{x+a}\right)$
154.  $\int \frac{x^3 dx}{(x^2-a^2)^2} = \frac{-a^2}{2(x^2-a^2)} + \frac{1}{2} \ln(x^2 - a^2)$
- 10 155.  $\int \frac{dx}{x(x^2-a^2)} = \frac{-1}{2a^2(x^2-a^2)} + \frac{1}{2a^4} \ln\left(\frac{x^2}{x^2-a^2}\right)$

156.  $\int \frac{dx}{x^2(x^2-a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2-a^2)} - \frac{3}{4a^6} \ln \left( \frac{x-a}{x+a} \right)$
157.  $\int \frac{dx}{x^3(x^2-a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2-a^2)} + \frac{1}{a^6} \ln \left( \frac{x^2}{x^2-a^2} \right)$
158.  $\int \frac{dx}{(x^2-a^2)^n} = \frac{-x}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(x^2-a^2)^{n-1}}$
159.  $\int \frac{x dx}{(x^2-a^2)^n} = \frac{-1}{2(n-1)(x^2-a^2)^{n-1}}$
160.  $\int \frac{dx}{x(x^2-a^2)^n} = \frac{-1}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{1}{a^2} \int \frac{dx}{x(x^2-a^2)^{n-1}}$
161.  $\int \frac{x^m dx}{(x^2-a^2)^n} = \int \frac{x^{m-2} dx}{(x^2-a^2)^{n-1}} + a^2 \int \frac{x^{m-2} dx}{(x^2-a^2)^n}$
162.  $\int \frac{dx}{x^m(x^2-a^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^{m-2}(x^2-a^2)^n} - \frac{1}{a^2} \int \frac{dx}{x^m(x^2-a^2)^{n-1}}$

INTEGRALES CON  $a^2-x^2$ ,  $x^2 < a^2$

163.  $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left( \frac{a+x}{a-x} \right) \text{ ó } \frac{1}{a} \operatorname{tgh}^{-1} \frac{x}{a}$
164.  $\int \frac{x dx}{a^2-x^2} = -\frac{1}{2} \ln(a^2-x^2)$
165.  $\int \frac{x^2 dx}{a^2-x^2} = -x + \frac{a}{2} \ln \left( \frac{a+x}{a-x} \right)$
166.  $\int \frac{x^3 dx}{a^2-x^2} = -\frac{x^2}{2} - \frac{a^2}{2} \ln(a^2-x^2)$
167.  $\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2} \ln \left( \frac{x^2}{a^2-x^2} \right)$
168.  $\int \frac{dx}{x^2(a^2-x^2)} = -\frac{1}{a^2x} + \frac{1}{2a^3} \ln \left( \frac{a+x}{a-x} \right)$
169.  $\int \frac{dx}{x^3(a^2-x^2)} = -\frac{1}{2a^2x^2} + \frac{1}{2a^4} \ln \left( \frac{x^2}{a^2-x^2} \right)$
170.  $\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^4(a^2-x^2)} + \frac{1}{4a^3} \ln \left( \frac{a+x}{a-x} \right)$
171.  $\int \frac{x dx}{(a^2-x^2)^2} = \frac{-1}{2(a^2-x^2)}$
172.  $\int \frac{x^2 dx}{(a^2-x^2)^2} = \frac{x}{2(a^2-x^2)} - \frac{1}{4a} \ln \left( \frac{a+x}{a-x} \right)$
173.  $\int \frac{x^3 dx}{(a^2-x^2)^2} = \frac{a^2}{2(a^2-x^2)} + \frac{1}{2} \ln(a^2-x^2)$
174.  $\int \frac{dx}{x(a^2-x^2)^2} = \frac{1}{2a^2(a^2-x^2)} + \frac{1}{2a^4} \ln \left( \frac{x^2}{a^2-x^2} \right)$
175.  $\int \frac{dx}{x^2(a^2-x^2)^2} = -\frac{1}{a^4x} + \frac{x}{2a^4(a^2-x^2)} + \frac{3}{4a^5} \ln \left( \frac{a+x}{a-x} \right)$
176.  $\int \frac{dx}{x^3(a^2-x^2)^2} = -\frac{1}{2a^4x^2} + \frac{1}{2a^4(a^2-x^2)} + \frac{1}{a^6} \ln \left( \frac{x^2}{a^2-x^2} \right)$
177.  $\int \frac{dx}{(a^2-x^2)^n} = \frac{x}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{dx}{(a^2-x^2)^{n-1}}$
178.  $\int \frac{x dx}{(a^2-x^2)^n} = \frac{1}{2(n-1)(a^2-x^2)^{n-1}}$
179.  $\int \frac{dx}{x(a^2-x^2)^n} = \frac{1}{2(n-1)a^2(a^2-x^2)^{n-1}}$
180.  $\int \frac{x^m dx}{(a^2-x^2)^n} = a^2 \int \frac{x^{m-2} dx}{(a^2-x^2)^n} - \int \frac{x^{m-2} dx}{(a^2-x^2)^{n-1}}$
181.  $\int \frac{dx}{x^m(a^2-x^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^{m-2}(a^2-x^2)^n} + \frac{1}{a^2} \int \frac{dx}{x^m(a^2-x^2)^{n-1}}$

INTEGRALES CON  $\sqrt{x^2+a^2}$

182.  $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2})$  ó  $\sinh^{-1} \frac{x}{a}$
183.  $\int \frac{x dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$
184.  $\int \frac{x^2 dx}{\sqrt{x^2+a^2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2} \ln(x + \sqrt{x^2+a^2})$
185.  $\int \frac{x^3 dx}{\sqrt{x^2+a^2}} = \frac{\sqrt{(x^2+a^2)^3}}{3} - a^2 \sqrt{x^2+a^2}$
186.  $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a} \ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
187.  $\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2 x}$
188.  $\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2 x^2} + \frac{1}{2a^3} \ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
189.  $\int \sqrt{x^2+a^2} dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2} \ln(x + \sqrt{x^2+a^2})$
190.  $\int x \sqrt{x^2+a^2} dx = \frac{\sqrt{(x^2+a^2)^3}}{3}$
191.  $\int x^2 \sqrt{x^2+a^2} dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} - \frac{a^2 x \sqrt{x^2+a^2}}{8} - \frac{a^4}{8} \ln(x + \sqrt{x^2+a^2})$
192.  $\int x^3 \sqrt{x^2+a^2} dx = \frac{\sqrt{(x^2+a^2)^5}}{5} - \frac{a^2 \sqrt{(x^2+a^2)^3}}{3}$
193.  $\int \frac{\sqrt{x^2+a^2}}{x} dx = \sqrt{x^2+a^2} - a \ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
194.  $\int \frac{\sqrt{x^2+a^2}}{x^2} dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln(x + \sqrt{x^2+a^2})$
195.  $\int \frac{\sqrt{x^2+a^2}}{x^3} dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a} \ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
196.  $\int \frac{dx}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2 \sqrt{x^2+a^2}}$
197.  $\int \frac{x dx}{\sqrt{(x^2+a^2)^3}} = \frac{-1}{\sqrt{x^2+a^2}}$
198.  $\int \frac{x^2 dx}{\sqrt{(x^2+a^2)^3}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln(x + \sqrt{x^2+a^2})$
199.  $\int \frac{x^3 dx}{\sqrt{(x^2+a^2)^3}} = \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$
200.  $\int \frac{dx}{x\sqrt{(x^2+a^2)^3}} = \frac{1}{a^2 \sqrt{x^2+a^2}} - \frac{1}{a^3} \ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
201.  $\int \frac{dx}{x^2 \sqrt{(x^2+a^2)^3}} = -\frac{\sqrt{x^2+a^2}}{a^4 x} - \frac{x}{a^4 \sqrt{x^2+a^2}}$
202.  $\int \frac{dx}{x^3 \sqrt{(x^2+a^2)^3}} = \frac{-1}{2a^2 x^2 \sqrt{x^2+a^2}} - \frac{3}{2a^4 \sqrt{x^2+a^2}} + \frac{3}{2a^5} \ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
203.  $\int \sqrt{(x^2+a^2)^3} dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} + \frac{3a^2 x \sqrt{x^2+a^2}}{8} + \frac{3a^4}{8} \ln(x + \sqrt{x^2+a^2})$
204.  $\int x \sqrt{(x^2+a^2)^3} dx = \frac{\sqrt{(x^2+a^2)^5}}{5}$
205.  $\int x^2 \sqrt{(x^2+a^2)^3} dx = \frac{x\sqrt{(x^2+a^2)^5}}{6} - \frac{a^2 x \sqrt{(x^2+a^2)^3}}{24} - \frac{a^4 x \sqrt{x^2+a^2}}{16} - \frac{a^6}{16} \ln(x + \sqrt{x^2+a^2})$
206.  $\int x^3 \sqrt{(x^2+a^2)^3} dx = \frac{\sqrt{(x^2+a^2)^7}}{7} - \frac{a^2 \sqrt{(x^2+a^2)^5}}{5}$
207.  $\int \frac{\sqrt{(x^2+a^2)^3}}{x} dx = \frac{\sqrt{(x^2+a^2)^3}}{3} + a^2 \sqrt{x^2+a^2} - a^3 \ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
208.  $\int \frac{\sqrt{(x^2+a^2)^3}}{x^2} dx = -\frac{\sqrt{(x^2+a^2)^3}}{x} + \frac{3x\sqrt{x^2+a^2}}{2} + \frac{3}{2} a^2 \ln(x + \sqrt{x^2+a^2})$

$$209. \int \frac{\sqrt{(x^2+a^2)^3}}{x^3} dx = -\frac{\sqrt{(x^2+a^2)^3}}{2x} + \frac{3\sqrt{x^2+a^2}}{2} - \frac{3}{2} a \ln \left( \frac{a+\sqrt{x^2+a^2}}{x} \right)$$

INTEGRALES CON  $\sqrt{x^2 - a^2}$

$$210. \int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x + \sqrt{x^2 - a^2})$$

$$211. \int \frac{x dx}{\sqrt{x^2-a^2}} = \sqrt{x^2 - a^2}$$

$$212. \int \frac{x^3 dx}{\sqrt{x^2-a^2}} = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 - a^2})$$

$$213. \int \frac{x^5 dx}{\sqrt{x^2-a^2}} = \frac{\sqrt{(x^2-a^2)^3}}{3} + a^2 \sqrt{x^2 - a^2}$$

$$214. \int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right|$$

$$215. \int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2 x}$$

$$216. \int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2 x^2} + \frac{1}{2a^3} \sec^{-1} \left| \frac{x}{a} \right|$$

$$217. \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 - a^2})$$

$$218. \int x\sqrt{x^2 - a^2} dx = \frac{\sqrt{(x^2-a^2)^3}}{3}$$

$$219. \int x^2\sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2-a^2}}{4} + \frac{a^2 x\sqrt{x^2-a^2}}{8} - \frac{a^4}{8} \ln(x + \sqrt{x^2 - a^2})$$

$$220. \int x^3\sqrt{x^2 - a^2} dx = \frac{\sqrt{(x^2-a^2)^5}}{5} + \frac{a^2\sqrt{(x^2-a^2)^3}}{3}$$

$$221. \int \frac{\sqrt{x^2-a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right|$$

$$222. \int \frac{\sqrt{x^2-a^2}}{x^2} dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln(x + \sqrt{x^2 - a^2})$$

$$223. \int \frac{\sqrt{x^2-a^2}}{x^3} dx = -\frac{\sqrt{x^2-a^2}}{2x^2} + \frac{1}{2a} \sec^{-1} \left| \frac{x}{a} \right|$$

$$224. \int \frac{dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}}$$

$$225. \int \frac{x dx}{\sqrt{(x^2-a^2)^3}} = \frac{-1}{\sqrt{x^2-a^2}}$$

$$226. \int \frac{x^3 dx}{\sqrt{(x^2-a^2)^3}} = \frac{-x}{\sqrt{x^2-a^2}} + \ln(x + \sqrt{x^2 - a^2})$$

$$227. \int \frac{x^5 dx}{\sqrt{(x^2-a^2)^3}} = \sqrt{x^2 - a^2} - \frac{a^2}{\sqrt{x^2-a^2}}$$

$$228. \int \frac{dx}{x\sqrt{(x^2-a^2)^3}} = \frac{-1}{a^2\sqrt{x^2-a^2}} - \frac{1}{a^3} \sec^{-1} \left| \frac{x}{a} \right|$$

$$229. \int \frac{dx}{x^2\sqrt{(x^2-a^2)^3}} = -\frac{\sqrt{x^2-a^2}}{a^4 x} - \frac{x}{a^4\sqrt{x^2-a^2}}$$

$$230. \int \frac{dx}{x^3\sqrt{(x^2-a^2)^3}} = \frac{1}{2a^2 x^2\sqrt{x^2-a^2}} - \frac{3}{2a^4\sqrt{x^2-a^2}} - \frac{3}{2a^5} \sec^{-1} \left| \frac{x}{a} \right|$$

$$231. \int \sqrt{(x^2 - a^2)^3} dx = \frac{x\sqrt{(x^2-a^2)^3}}{4} - \frac{3a^2 x\sqrt{x^2-a^2}}{8} + \frac{3a^4}{8} \ln(x + \sqrt{x^2 - a^2})$$

$$232. \int x\sqrt{(x^2 - a^2)^3} dx = \frac{\sqrt{(x^2-a^2)^5}}{5}$$

$$233. \int x^2\sqrt{(x^2 - a^2)^3} dx = \frac{x\sqrt{(x^2-a^2)^5}}{6} + \frac{a^2 x\sqrt{(x^2-a^2)^3}}{24} - \frac{a^4 x\sqrt{x^2-a^2}}{16} + \frac{a^6}{16} \ln(x + \sqrt{x^2 - a^2})$$

$$234. \int x^3\sqrt{(x^2 - a^2)^3} dx = \frac{\sqrt{(x^2-a^2)^7}}{7} + \frac{a^2\sqrt{(x^2-a^2)^5}}{5}$$

$$235. \int \frac{\sqrt{(x^2-a^2)^3}}{x} dx = \frac{\sqrt{(x^2-a^2)^3}}{3} - a^2 \sqrt{x^2-a^2} + a^3 \sec^{-1} \left| \frac{x}{a} \right|$$

$$236. \int \frac{\sqrt{(x^2-a^2)^3}}{x^2} dx = -\frac{\sqrt{(x^2-a^2)^3}}{x} + \frac{3x\sqrt{x^2-a^2}}{2} - \frac{3}{2} a^2 \ln(x + \sqrt{x^2-a^2})$$

$$237. \int \frac{\sqrt{(x^2-a^2)^3}}{x^3} dx = -\frac{\sqrt{(x^2-a^2)^3}}{2x^2} + \frac{3\sqrt{x^2-a^2}}{2} - \frac{3}{2} a \sec^{-1} \left| \frac{x}{a} \right|$$

INTEGRALES CON  $\sqrt{a^2-x^2}$

$$238. \int \frac{dx}{\sqrt{a^2-x^2}} = \text{sen}^{-1} \frac{x}{a}$$

$$239. \int \frac{x dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$$

$$240. \int \frac{x^2 dx}{\sqrt{a^2-x^2}} = -\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \text{sen}^{-1} \frac{x}{a}$$

$$241. \int \frac{x^3 dx}{\sqrt{a^2-x^2}} = \frac{\sqrt{(a^2-x^2)^3}}{3} - a^2 \sqrt{a^2-x^2}$$

$$242. \int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a} \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right)$$

$$243. \int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2 x}$$

$$244. \int \frac{dx}{x^3\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{2a^2 x^2} - \frac{1}{2a^3} \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right)$$

$$245. \int \sqrt{a^2-x^2} dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \text{sen}^{-1} \frac{x}{a}$$

$$246. \int x\sqrt{a^2-x^2} dx = -\frac{\sqrt{(a^2-x^2)^3}}{3}$$

$$247. \int x^2\sqrt{a^2-x^2} dx = -\frac{x\sqrt{(a^2-x^2)^3}}{4} + \frac{a^2 x\sqrt{a^2-x^2}}{8} - \frac{a^4}{8} \text{sen}^{-1} \frac{x}{a}$$

$$248. \int x^3\sqrt{a^2-x^2} dx = \frac{\sqrt{(a^2-x^2)^5}}{5} - \frac{a^2\sqrt{(a^2-x^2)^3}}{3}$$

$$249. \int \frac{\sqrt{a^2-x^2}}{x} dx = \sqrt{a^2-x^2} - a \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right)$$

$$250. \int \frac{\sqrt{a^2-x^2}}{x^2} dx = -\frac{\sqrt{a^2-x^2}}{x} - \text{sen}^{-1} \frac{x}{a}$$

$$251. \int \frac{\sqrt{a^2-x^2}}{x^3} dx = -\frac{\sqrt{a^2-x^2}}{2x^2} + \frac{1}{2a} \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right)$$

$$252. \int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$

$$253. \int \frac{x dx}{\sqrt{(a^2-x^2)^3}} = \frac{1}{\sqrt{a^2-x^2}}$$

$$254. \int \frac{x^2 dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{\sqrt{a^2-x^2}} - \text{sen}^{-1} \frac{x}{a}$$

$$255. \int \frac{x^3 dx}{\sqrt{(a^2-x^2)^3}} = \sqrt{a^2-x^2} + \frac{a^2}{\sqrt{a^2-x^2}}$$

$$256. \int \frac{dx}{x\sqrt{(a^2-x^2)^3}} = \frac{1}{a^2\sqrt{a^2-x^2}} - \frac{1}{a^3} \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right)$$

$$257. \int \frac{dx}{x^2\sqrt{(a^2-x^2)^3}} = -\frac{\sqrt{a^2-x^2}}{a^4 x} + \frac{x}{a^4\sqrt{a^2-x^2}}$$

$$258. \int \frac{dx}{x^3\sqrt{(a^2-x^2)^3}} = \frac{-1}{2a^2 x^2\sqrt{a^2-x^2}} + \frac{3}{2a^4\sqrt{a^2-x^2}} - \frac{3}{2a^6} \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right)$$

$$259. \int \sqrt{(a^2-x^2)^3} dx = \frac{x\sqrt{(a^2-x^2)^3}}{4} + \frac{3a^2 x\sqrt{a^2-x^2}}{8} + \frac{3a^4}{8} \text{sen}^{-1} \frac{x}{a}$$

$$\begin{aligned}
260. \int x \sqrt{(a^2 - x^2)^3} dx &= -\frac{\sqrt{(a^2 - x^2)^5}}{5} \\
261. \int x^2 \sqrt{(a^2 - x^2)^3} dx &= -\frac{x \sqrt{(a^2 - x^2)^5}}{6} + \frac{a^2 x \sqrt{(a^2 - x^2)^3}}{24} + \frac{a^4 x \sqrt{a^2 - x^2}}{16} + \frac{a^6}{16} \operatorname{sen}^{-1} \frac{x}{a} \\
262. \int x^3 \sqrt{(a^2 - x^2)^3} dx &= \frac{\sqrt{(a^2 - x^2)^7}}{7} - \frac{a^2 \sqrt{(a^2 - x^2)^5}}{5} \\
263. \int \frac{\sqrt{(a^2 - x^2)^3}}{x} dx &= \frac{\sqrt{(a^2 - x^2)^3}}{3} + a^2 \sqrt{a^2 - x^2} - a^3 \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) \\
264. \int \frac{\sqrt{(a^2 - x^2)^3}}{x^2} dx &= -\frac{\sqrt{(a^2 - x^2)^3}}{x} - \frac{3x \sqrt{a^2 - x^2}}{2} - \frac{3}{2} a^2 \operatorname{sen}^{-1} \frac{x}{a} \\
265. \int \frac{\sqrt{(a^2 - x^2)^3}}{x^3} dx &= -\frac{\sqrt{(a^2 - x^2)^3}}{2x^2} - \frac{3\sqrt{a^2 - x^2}}{2} + \frac{3}{2} a \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right)
\end{aligned}$$

**INTEGRALES CON  $ax^2 + bx + c$**

Si  $b^2 = 4ac$ , se puede escribir  $ax^2 + bx + c = a(x + b/2a)^2$  y se emplean los resultados de las páginas 11 y 12.

$$\begin{aligned}
266. \int \frac{dx}{ax^2 + bx + c} &= \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \operatorname{tg}^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} \\ \frac{1}{\sqrt{b^2 - 4ac}} \ln \left( \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right) \end{cases} \\
267. \int \frac{x dx}{ax^2 + bx + c} &= \frac{1}{2a} \ln(ax^2 + bx + c) - \frac{b}{2a} \int \frac{dx}{ax^2 + bx + c} \\
268. \int \frac{x^2 dx}{ax^2 + bx + c} &= \frac{x}{a} - \frac{b}{2a^2} \ln(ax^2 + bx + c) + \frac{b^2 - 2ac}{2a^2} \int \frac{dx}{ax^2 + bx + c} \\
269. \int \frac{x^m dx}{ax^2 + bx + c} &= \frac{x^{m-1}}{(m-1)a} - \frac{c}{a} \int \frac{x^{m-2} dx}{ax^2 + bx + c} - \frac{b}{a} \int \frac{x^{m-1} dx}{ax^2 + bx + c} \\
270. \int \frac{dx}{x(ax^2 + bx + c)} &= \frac{1}{2c} \ln \left( \frac{x^2}{ax^2 + bx + c} \right) - \frac{b}{2c} \int \frac{dx}{ax^2 + bx + c} \\
271. \int \frac{dx}{x^2(ax^2 + bx + c)} &= \frac{b}{2c^2} \ln \left( \frac{ax^2 + bx + c}{x^2} \right) - \frac{1}{xc} + \frac{b^2 - 2ac}{2c^2} \int \frac{dx}{ax^2 + bx + c} \\
272. \int \frac{dx}{x^n(ax^2 + bx + c)} &= -\frac{1}{(n-1)cx^{n-1}} - \frac{b}{c} \int \frac{dx}{x^{n-1}(ax^2 + bx + c)} - \frac{a}{c} \int \frac{dx}{x^{n-2}(ax^2 + bx + c)} \\
273. & \int \text{NO SE ENTIENDE NADA DE LO QUE DICE} \\
274. \int \frac{x dx}{(ax^2 + bx + c)^2} &= -\frac{bx + 2c}{(4ac - b^2)(ax^2 + bx + c)} - \frac{b}{4ac - b^2} \int \frac{dx}{ax^2 + bx + c} \\
275. \int \frac{x^2 dx}{(ax^2 + bx + c)^2} &= \frac{(b^2 - 2ac)x + bc}{a(4ac - b^2)(ax^2 + bx + c)} + \frac{2c}{4ac - b^2} \int \frac{dx}{ax^2 + bx + c} \\
276. \int \frac{x^m dx}{(ax^2 + bx + c)^n} &= \frac{-x^{m-1}}{(2n-m-1)a(ax^2 + bx + c)^{n-1}} + \frac{1}{(2n-m-1)a} \left\{ c \int \frac{(m-1)x^{m-2} dx}{(ax^2 + bx + c)^n} - b \int \frac{(n-m)x^{m-1} dx}{(ax^2 + bx + c)^n} \right\} \\
277. \int \frac{x^{2n-1} dx}{(ax^2 + bx + c)^n} &= \frac{1}{a} \int \frac{x^{2n-3} dx}{(ax^2 + bx + c)^{n-1}} - \frac{c}{a} \int \frac{x^{2n-3} dx}{(ax^2 + bx + c)^n} - \frac{b}{a} \int \frac{x^{2n-2} dx}{(ax^2 + bx + c)^n} \\
278. \int \frac{dx}{x(ax^2 + bx + c)^2} &= \frac{1}{2c(ax^2 + bx + c)} - \frac{b}{2c} \int \frac{dx}{(ax^2 + bx + c)^2} + \frac{1}{c} \int \frac{dx}{x(ax^2 + bx + c)} \\
279. \int \frac{dx}{x^2(ax^2 + bx + c)^2} &= \frac{-1}{cx(ax^2 + bx + c)} - \frac{3a}{c} \int \frac{dx}{(ax^2 + bx + c)^2} - \frac{2b}{c} \int \frac{dx}{x(ax^2 + bx + c)^2} \\
280. \int \frac{dx}{x^m(ax^2 + bx + c)^n} &= \frac{-1}{(m-1)cx^{m-1}(ax^2 + bx + c)^{n-1}} - \frac{(m+2n-3)a}{(m-1)c} \int \frac{dx}{x^{m-2}(ax^2 + bx + c)^n} - \frac{(m+n-2)b}{(m-1)c} \int \frac{dx}{x^{m-1}(ax^2 + bx + c)^n}
\end{aligned}$$

**INTEGRALES CON  $\sqrt{ax^2 + bx + c}$**

Si en las fórmulas siguientes  $b^2 = 4ac$ , se puede escribir  $\sqrt{ax^2 + bx + c} = \sqrt{a}(x + \frac{b}{2a})$  y se emplean los resultados de las páginas 11 y 12.

281.  $\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \begin{cases} \frac{1}{\sqrt{a}} \ln(2\sqrt{a}\sqrt{(ax^2 + bx + c)} + 2ax + b) \\ -\frac{1}{\sqrt{-a}} \operatorname{sen}^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) \text{ ó } \frac{1}{\sqrt{a}} \operatorname{senh}^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) \end{cases}$
282.  $\frac{xdx}{\sqrt{ax^2 + bx + c}} = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
283.  $\int \frac{x^2 dx}{\sqrt{ax^2 + bx + c}} = \frac{2ax-3b}{4a^2} \sqrt{ax^2 + bx + c} + \frac{3b^2-4ac}{8a^2} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
284.  $\int \frac{dx}{x\sqrt{ax^2 + bx + c}} = \begin{cases} -\frac{1}{\sqrt{c}} \ln\left(\frac{2\sqrt{c}\sqrt{ax^2 + bx + c} + bx + 2c}{x}\right) \\ \frac{1}{\sqrt{-c}} \operatorname{sen}^{-1}\left(\frac{bx+2c}{|x|\sqrt{b^2-4ac}}\right) \text{ ó } -\frac{1}{\sqrt{c}} \operatorname{senh}^{-1}\left(\frac{bx+2c}{|x|\sqrt{4ac-b^2}}\right) \end{cases}$
285.  $\int \frac{dx}{x^2\sqrt{ax^2 + bx + c}} = -\frac{\sqrt{ax^2 + bx + c}}{ax} - \frac{b}{2c} \int \frac{dx}{x\sqrt{ax^2 + bx + c}}$
286.  $\int \sqrt{ax^2 + bx + c} dx = \frac{2ax+b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac-b^2}{8a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
287.  $\int x\sqrt{ax^2 + bx + c} dx = \frac{\sqrt{(ax^2 + bx + c)^3}}{3a} - \frac{b(2ax+b)}{8a^2} \sqrt{ax^2 + bx + c} - \frac{b(4ac-b^2)}{16a^2} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
288.  $\int x^2\sqrt{ax^2 + bx + c} dx = \frac{6ax-5b}{24a^2} \sqrt{(ax^2 + bx + c)^3} + \frac{5b^2-4ac}{16a^2} \int \sqrt{ax^2 + bx + c} dx$
289.  $\int \frac{\sqrt{ax^2 + bx + c}}{x} dx = \frac{\sqrt{ax^2 + bx + c}}{x} + \frac{b}{2} \int \frac{dx}{\sqrt{ax^2 + bx + c}} + c \int \frac{dx}{x\sqrt{ax^2 + bx + c}}$
290.  $\int \frac{\sqrt{ax^2 + bx + c}}{x^2} dx = -\frac{\sqrt{ax^2 + bx + c}}{x} + a \int \frac{dx}{\sqrt{ax^2 + bx + c}} + \frac{b}{2} \int \frac{dx}{x\sqrt{ax^2 + bx + c}}$
291.  $\int \frac{dx}{\sqrt{(ax^2 + bx + c)^3}} = \frac{2(2ax+b)}{(4ac-b^2)\sqrt{ax^2 + bx + c}}$
292.  $\int \frac{xdx}{\sqrt{(ax^2 + bx + c)^3}} = \frac{2(bx+2c)}{(b^2-4ac)\sqrt{ax^2 + bx + c}}$
293.  $\int \frac{x^2 dx}{\sqrt{(ax^2 + bx + c)^3}} = \frac{(2b^2-4ac)x+2bc}{a(4ac)\sqrt{ax^2 + bx + c}} + \frac{1}{a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
294.  $\int \frac{dx}{x\sqrt{(ax^2 + bx + c)^3}} = \frac{1}{c\sqrt{ax^2 + bx + c}} + \frac{1}{c} \int \frac{dx}{x\sqrt{ax^2 + bx + c}} - \frac{b}{2c} \int \frac{dx}{\sqrt{(ax^2 + bx + c)^3}}$
295.  $\int \frac{dx}{x^2\sqrt{(ax^2 + bx + c)^3}} = \frac{-(ax^2 + bx + c)}{c^2x\sqrt{ax^2 + bx + c}} - \frac{3b}{2c^2} \int \frac{dx}{x\sqrt{ax^2 + bx + c}} + \frac{b^2-4ac}{2c^2} \int \frac{dx}{\sqrt{(ax^2 + bx + c)^3}}$
296.  $\int \sqrt{(ax^2 + bx + c)^{n+1}} dx = \frac{(2ax+b)\sqrt{(ax^2 + bx + c)^{n+1}}}{4a(n+1)} + \frac{(2n+1)(4ac-b^2)}{8a(n+1)} \int \sqrt{(ax^2 + bx + c)^{n-1}} dx$
297.  $\int x\sqrt{(ax^2 + bx + c)^{n+1}} dx = \frac{\sqrt{(ax^2 + bx + c)^{n+3}}}{a(2n+3)} - \frac{b}{2a} \int \sqrt{(ax^2 + bx + c)^{n+1}} dx$
298.  $\int \frac{dx}{\sqrt{(ax^2 + bx + c)^{n+1}}} dx = \frac{1}{(2n-1)(4ac-b^2)} \left( \frac{2(2ax+b)}{\sqrt{(ax^2 + bx + c)^{n-1}}} + 8a(n-1) \int \frac{dx}{\sqrt{(ax^2 + bx + c)^{n-1}}} \right)$
- 299.

### INTEGRALES CON $x^3 + a^3$

Para integrales con  $x^3 - a^3$ , se reemplaza a por -a

300.  $\int \frac{dx}{x^3 + a^3} = \frac{1}{6a^2} \ln \left| \frac{(x+a)^2}{x^2 - ax + a^2} \right| + \frac{1}{a^2\sqrt{3}} \operatorname{tg}^{-1} \left( \frac{2x-a}{a\sqrt{3}} \right)$

301.  $\int \frac{xdx}{x^3 + a^3} = \frac{1}{6a} \ln \left| \frac{x^2 - ax + a^2}{(x+a)^2} \right| + \frac{1}{a\sqrt{3}} \operatorname{tg}^{-1} \left( \frac{2x-a}{a\sqrt{3}} \right)$

302.  $\int \frac{x^2 dx}{x^3 + a^3} = \frac{1}{3} \ln(x^3 + a^3)$

16 303.  $\int \frac{dx}{x(x^3 + a^3)} = \frac{1}{3a^3} \ln \left( \frac{x^3}{x^3 + a^3} \right)$

304.  $\int \frac{dx}{x^2(x^3+a^3)} = -\frac{1}{a^3x} - \frac{1}{6a^4} \ln \frac{x^2-ax+a^2}{(x+a)^2} - \frac{1}{a^4\sqrt{3}} \operatorname{tg}^{-1} \left( \frac{2x-a}{a\sqrt{3}} \right)$
305.  $\int \frac{dx}{(x^3+a^3)^2} = \frac{x}{3a^3(x^3+a^3)} + \frac{1}{9a^3} \ln \left( \frac{[x+a]^2}{x^2-ax+a^2} \right) + \frac{2}{3a^5\sqrt{3}} \operatorname{tg}^{-1} \left( \frac{2x-a}{a\sqrt{3}} \right)$
306.  $\int \frac{x dx}{(x^3+a^3)^2} = \frac{x^2}{3a^3(x^3+a^3)} + \frac{1}{18a^4} \ln \left( \frac{x^2-ax+a^2}{[x+a]^2} \right) + \frac{1}{3a^4\sqrt{3}} \operatorname{tg}^{-1} \left( \frac{2x-a}{a\sqrt{3}} \right)$
307.  $\int \frac{x^2 dx}{(x^3+a^3)^2} = -\frac{1}{3(x^3+a^3)}$
308.  $\int \frac{dx}{x(x^3+a^3)^2} = \frac{1}{3a^3(x^3+a^3)} + \frac{1}{3a^6} \ln \left( \frac{x^3}{x^3+a^3} \right)$
309.  $\int \frac{dx}{x^2(x^3+a^3)^2} = -\frac{1}{a^6x} - \frac{x^2}{3a^6(x^3+a^3)} - \frac{4}{3a^6} \int \frac{x dx}{x^3+a^3}$  Véase 301
310.  $\int \frac{x^m dx}{x^3+a^3} = \frac{x^{m-2}}{(m-2)} - a^3 \int \frac{x^{m-3} dx}{x^3+a^3}$
311.  $\int \frac{dx}{x^n(x^3+a^3)} = \frac{-1}{a^3(n-1)x^{n-1}} - \frac{1}{a^3} \int \frac{dx}{x^{n-3}(x^3+a^3)}$

INTEGRALES CON  $x^4 \pm a^4$

312.  $\int \frac{dx}{x^4+a^4} = \frac{1}{4a^3\sqrt{2}} \ln \left( \frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2} \right) - \frac{1}{2a^3\sqrt{2}} \operatorname{tg}^{-1} \left( \frac{ax\sqrt{2}}{x^2-a^2} \right)$
313.  $\int \frac{x dx}{x^4+a^4} = \frac{1}{2a^2} \operatorname{tg}^{-1} \left( \frac{x^2}{a^2} \right)$
314.  $\int \frac{x^2 dx}{x^4+a^4} = \frac{1}{4a\sqrt{2}} \ln \left( \frac{x^2-ax\sqrt{2}+a^2}{x^2+ax\sqrt{2}+a^2} \right) - \frac{1}{2a\sqrt{2}} \operatorname{tg}^{-1} \left( \frac{ax\sqrt{2}}{x^2-a^2} \right)$
315.  $\int \frac{x^3 dx}{x^4+a^4} = \frac{1}{4} \ln(x^4+a^4)$
316.  $\int \frac{dx}{x(x^4+a^4)} = \frac{1}{4a^4} \ln \left( \frac{x^4}{x^4+a^4} \right)$
317.  $\int \frac{dx}{x^2(x^4+a^4)} = -\frac{1}{a^4x} - \frac{1}{4a^5\sqrt{2}} \ln \left( \frac{x^2-ax\sqrt{2}+a^2}{x^2+ax\sqrt{2}+a^2} \right) + \frac{1}{2a^5\sqrt{2}} \operatorname{tg}^{-1} \left( \frac{ax\sqrt{2}}{x^2-a^2} \right)$
318.  $\int \frac{dx}{x^3(x^4+a^4)} = -\frac{1}{2a^4x^2} - \frac{1}{2a^5} \operatorname{tg}^{-1} \left( \frac{x^2}{a^2} \right)$
319.  $\int \frac{dx}{x^4-a^4} = \frac{1}{4a^3} \ln \left( \frac{x-a}{x+a} \right) - \frac{1}{2a^3} \operatorname{tg}^{-1} \left( \frac{x}{a} \right)$
320.  $\int \frac{x dx}{x^4-a^4} = \frac{1}{4a^2} \ln \left( \frac{x^2-a^2}{x^2+a^2} \right)$
321.  $\int \frac{x^2 dx}{x^4-a^4} = \frac{1}{4a} \ln \left( \frac{x-a}{x+a} \right) + \frac{1}{2a} \operatorname{tg}^{-1} \left( \frac{x}{a} \right)$
322.  $\int \frac{x^3 dx}{x^4-a^4} = \frac{1}{4} \ln(x^4-a^4)$
323.  $\int \frac{dx}{x(x^4-a^4)} = \frac{1}{4a^4} \ln \left( \frac{x^4-a^4}{x^4} \right)$
324.  $\int \frac{dx}{x^2(x^4-a^4)} = \frac{1}{a^4x} + \frac{1}{4a^5} \ln \left( \frac{x-a}{x+a} \right) + \frac{1}{2a^5} \operatorname{tg}^{-1} \left( \frac{x}{a} \right)$
325.  $\int \frac{dx}{x^3(x^4-a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6} \ln \left( \frac{x^2-a^2}{x^2+a^2} \right)$

INTEGRALES CON  $x^n \pm a^n$

326.  $\int \frac{dx}{x(x^n+a^n)} = \frac{1}{na^n} \ln \left( \frac{x^n}{x^n+a^n} \right)$
327.  $\int \frac{x^{n-1} dx}{x^n+a^n} = \frac{1}{n} \ln(x^n+a^n)$

$$328. \int \frac{x^m dx}{(x^r + a^r)^n} = \int \frac{x^{m-r} dx}{(x^r + a^r)^{n-1}} - \frac{1}{a^r} \int \frac{dx}{x^{m-r} (x^r + a^r)^n}$$

$$329. \int \frac{dx}{x^m (x^r + a^r)^n} = \frac{1}{a^r} \int \frac{dx}{x^{m-r} (x^r + a^r)^{n-1}} - \frac{1}{a^r} \int \frac{dx}{x^{m-r} (x^r + a^r)^n}$$

$$330. \int \frac{dx}{x \sqrt{x^n + a^n}} = \frac{1}{n \sqrt{a^n}} \ln \left( \frac{\sqrt{x^n + a^n} - \sqrt{a^n}}{\sqrt{x^n + a^n} + \sqrt{a^n}} \right)$$

$$331. \int \frac{dx}{x(x^n - a^n)} = \frac{1}{na^n} \ln \left( \frac{x^n - a^n}{x^n} \right)$$

$$332. \int \frac{x^{n-1} dx}{x^n - a^n} = \frac{1}{n} \ln(x^n - a^n)$$

$$333. \int \frac{x^m dx}{(x^r - a^r)^n} = a^r \int \frac{x^{m-r} dx}{(x^r - a^r)^n} + \int \frac{x^{m-r} dx}{(x^r - a^r)^{n-1}}$$

$$334. \int \frac{dx}{x^m (x^r - a^r)^n} = \frac{1}{a^r} \int \frac{dx}{x^{m-r} (x^r - a^r)^n} - \frac{1}{a^r} \int \frac{dx}{x^{m-r} (x^r - a^r)^{n-1}}$$

$$335. \int \frac{dx}{x \sqrt{x^n - a^n}} = \frac{2}{n \sqrt{a^n}} \cos^{-1} \sqrt{\frac{a^n}{x^n}}$$

$$336. \int \frac{x^{p-1} dx}{(x^{2m} + a^{2m})} = \frac{1}{ma^{2m-p}} \sum_{k=1}^m \operatorname{sen} \left( \frac{(2k-1)p\pi}{2m} \right) \cdot \operatorname{tg}^{-1} \left( \frac{x+a \cos \left[ \frac{(2k-1)\pi}{2m} \right]}{a \operatorname{sen} \left[ \frac{(2k-1)\pi}{2m} \right]} \right) - \\ - \frac{1}{2ma^{2m-p}} \sum_{k=1}^m \cos \left( \frac{(2k-1)p\pi}{2m} \right) \cdot \ln \{ x^2 + 2ax \cos \left( \frac{(2k-1)\pi}{2m} \right) + a^2 \}$$

$$337. \int \frac{x^{p-1} dx}{(x^{2m} - a^{2m})} = \frac{1}{2ma^{2m-p}} \sum_{k=1}^{m-1} \cos \frac{k p \pi}{m} \cdot \ln \{ x^2 - 2ax \cos \left( \frac{k\pi}{m} \right) + a^2 \} - \\ - \frac{1}{ma^{2m-p}} \sum_{k=1}^{m-1} \operatorname{sen} \frac{k p \pi}{m} \cdot \operatorname{tg}^{-1} \left( \frac{x-a \cos \frac{k\pi}{m}}{a \operatorname{sen} \frac{k\pi}{m}} \right) + \\ + \frac{1}{2ma^{2m-p}} \{ \ln(x-a) + (-1)^p \ln(x+a) \}$$

tanto en 336 como 337 es  $0 < p \leq 2m$

$$338. \int \frac{x^{p-1} dx}{x^{2m+1} - a^{2m+1}} = \frac{2(-1)^{p-1}}{(2m+1)a^{2m-p+1}} \sum_{k=1}^m \operatorname{sen} \frac{2kp\pi}{2m+1} \cdot \operatorname{tg}^{-1} \left( \frac{x+a \cos \frac{2k\pi}{2m+1}}{a \operatorname{sen} \frac{2k\pi}{2m+1}} \right) - \\ - \frac{(-1)^{p-1}}{(2m+1)a^{2m-p+1}} \sum_{k=1}^m \cos \frac{2kp\pi}{2m+1} \cdot \ln \{ x^2 - 2ax \cos \frac{2k\pi}{2m+1} + a^2 \} + \\ + \frac{(-1)^{p-1} \ln(x+a)}{(2m+1)a^{2m-p+1}}$$

$$339. \int \frac{x^{p-1} dx}{x^{2m+1} - a^{2m+1}} = \frac{-2}{(2m+1)a^{2m-p+1}} \sum_{k=1}^m \operatorname{sen} \frac{2kp\pi}{2m+1} \cdot \operatorname{tg}^{-1} \left( \frac{x-a \cos \frac{2k\pi}{2m+1}}{a \operatorname{sen} \frac{2k\pi}{2m+1}} \right) + \\ + \frac{1}{(2m+1)a^{2m-p+1}} \sum_{k=1}^m \cos \frac{2kp\pi}{2m+1} \cdot \ln \{ x^2 - 2ax \cos \frac{2k\pi}{2m+1} + a^2 \} + \\ + \frac{\ln(x-a)}{(2m+1)a^{2m-p+1}}$$

tanto en 338 como en 339 es  $0 < p \leq 2m+1$

INTEGRALES CON  $\operatorname{sen} ax$

340.  $\int \operatorname{sen} ax \, dx = -\frac{\cos ax}{a}$
341.  $\int x \operatorname{sen} ax \, dx = \frac{\operatorname{sen} ax}{a^2} - \frac{x \cos ax}{a}$
342.  $\int x^2 \operatorname{sen} ax \, dx = \frac{2x}{a^2} \operatorname{sen} ax + \left(\frac{2}{a^3} - \frac{x^2}{a}\right) \cos ax$
343.  $\int x^3 \operatorname{sen} ax \, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right) \operatorname{sen} ax + \left(\frac{6x}{a^3} - \frac{x^3}{a}\right) \cos ax$
344.  $\int \frac{\operatorname{sen} ax}{x} \, dx = ax - \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (ax)^{2n-1}}{(2n-1)(2n-1)!}$
345.  $\int \frac{\operatorname{sen} ax}{x^2} \, dx = -\frac{\operatorname{sen} ax}{x} + a \int \frac{\cos ax}{x} \, dx$  Véase 374
346.  $\int \frac{dx}{\operatorname{sen} ax} = \frac{1}{a} \ln(\operatorname{cosec} ax - \cot g ax) = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2}$
347.  $\int \frac{xdx}{\operatorname{sen} ax} = \frac{1}{a^2} \left\{ ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \dots + \frac{2(2^{n-1}-1)B_n(ax)^{2n-1}}{(2n+1)!} + \dots \right\}$   $B_n$  es  $n^{\circ}$  de Bernoulli
348.  $\int \operatorname{sen}^2 ax \, dx = \frac{x}{2} - \frac{\operatorname{sen} ax}{4a}$
349.  $\int x \operatorname{sen}^2 ax \, dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2ax}{4a} - \frac{\cos 2ax}{8a^2}$
350.  $\int \operatorname{sen}^3 ax \, dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a}$
351.  $\int \operatorname{sen}^4 ax \, dx = \frac{3x}{8} - \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a}$
352.  $\int \frac{dx}{\operatorname{sen}^2 ax} = -\frac{1}{a} \cot g ax$
353.  $\int \frac{dx}{\operatorname{sen}^3 ax} = -\frac{\cos ax}{2a \operatorname{sen}^2 ax} + \frac{1}{2a} \ln \operatorname{tg} \frac{ax}{2}$
354.  $\int \operatorname{sen} px \operatorname{sen} qx \, dx = \frac{\operatorname{sen}(p-q)x}{2(p-q)} - \frac{\operatorname{sen}(p+q)x}{2(p+q)}$  Si  $p = \pm q$ , véase 348
355.  $\int \frac{dx}{1-\operatorname{sen} ax} = \frac{1}{a} \operatorname{tg} \left[ \frac{\pi}{4} + \frac{ax}{2} \right]$
356.  $\int \frac{xdx}{1-\operatorname{sen} ax} = \frac{x}{a} \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right) + \frac{2}{a^2} \ln \operatorname{sen} \left( \frac{\pi}{4} - \frac{ax}{2} \right)$
357.  $\int \frac{dx}{1+\operatorname{sen} ax} = -\frac{1}{a} \operatorname{tg} \left( \frac{\pi}{4} - \frac{ax}{2} \right)$
358.  $\int \frac{xdx}{1+\operatorname{sen} ax} = -\frac{x}{a} \operatorname{tg} \left( \frac{\pi}{4} - \frac{ax}{2} \right) + \frac{2}{a^2} \ln \operatorname{sen} \left( \frac{\pi}{4} + \frac{ax}{2} \right)$
359.  $\int \frac{dx}{(1-\operatorname{sen} ax)^2} = \frac{1}{2a} \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right) + \frac{1}{6a} \operatorname{tg}^3 \left( \frac{\pi}{4} + \frac{ax}{2} \right)$
360.  $\int \frac{dx}{(1+\operatorname{sen} ax)^2} = -\frac{1}{2a} \operatorname{tg} \left( \frac{\pi}{4} - \frac{ax}{2} \right) - \frac{1}{6a} \operatorname{tg}^3 \left( \frac{\pi}{4} - \frac{ax}{2} \right)$
361.  $\int \frac{dx}{p+q \operatorname{sen} ax} = \begin{cases} \frac{2}{a\sqrt{p^2-q^2}} \operatorname{tg}^{-1} \left( \frac{p \operatorname{tg} \frac{ax}{2} + q}{\sqrt{p^2-q^2}} \right) \\ \frac{1}{a\sqrt{q^2-p^2}} \ln \left( \frac{p \operatorname{tg} \frac{ax}{2} + q - \sqrt{q^2-p^2}}{p \operatorname{tg} \frac{ax}{2} + q + \sqrt{q^2-p^2}} \right) \end{cases}$  Si  $p = \pm q$ , Véase 355 y 357
362.  $\int \frac{dx}{(p+q \operatorname{sen} ax)^2} = \frac{q \cos ax}{a(p^2-q^2)(p+q \operatorname{sen} ax)} + \frac{p}{(p^2-q^2)} \int \frac{dx}{p+q \operatorname{sen} ax}$  Si  $p = \pm q$ , véase 359 y 360

$$363. \int \frac{dx}{p^2+q^2 \operatorname{sen}^2 ax} = \frac{1}{ap \sqrt{p^2+q^2}} \operatorname{tg}^{-1} \left( \frac{\sqrt{p^2+q^2} \operatorname{tg} ax}{p} \right)$$

$$364. \int \frac{dx}{p^2-q^2 \operatorname{sen}^2 ax} = \begin{cases} \frac{1}{ap \sqrt{p^2-q^2}} \operatorname{tg}^{-1} \left( \frac{\sqrt{p-q} \operatorname{tg} ax}{p} \right) \\ \frac{1}{2ap \sqrt{q^2-p^2}} \ln \left( \frac{\sqrt{q^2-p^2} \operatorname{tg} ax + p}{\sqrt{q^2-p^2} \operatorname{tg} ax - p} \right) \end{cases}$$

$$365. \int x^m \operatorname{sen} ax \, dx = -\frac{x^m \cos ax}{a} + \frac{mx^{m-1} \operatorname{sen} ax}{a^2} - \frac{m(m-1)}{a^3} \int x^{m-2} \operatorname{sen} ax \, dx$$

$$366. \int \frac{\operatorname{sen} ax}{x^n} dx = -\frac{\operatorname{sen} ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\cos ax}{x^{n-1}} dx \quad \text{Véase 396}$$

$$367. \int \operatorname{sen}^n ax \, dx = -\frac{\operatorname{sen}^{n-1} ax \cos ax}{an} + \frac{n-1}{n} \int \operatorname{sen}^{n-2} ax \, dx$$

$$368. \int \frac{dx}{\operatorname{sen}^n ax} = \frac{-\cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\operatorname{sen}^{n-2} ax}$$

$$369. \int \frac{x dx}{\operatorname{sen}^n ax} = \frac{-x \cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\operatorname{sen}^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x dx}{\operatorname{sen}^{n-2} ax}$$

### INTEGRALES CON $\cos ax$

$$370. \int \cos ax \, dx = \frac{\operatorname{sen} ax}{a}$$

$$371. \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \operatorname{sen} ax}{a}$$

$$372. \int x^2 \cos ax \, dx = \frac{2x}{a^2} \cos ax + \left( \frac{x^2}{a} - \frac{2}{a^3} \right) \operatorname{sen} ax$$

$$373. \int x^3 \cos ax \, dx = \left( \frac{3x^2}{a^2} - \frac{6}{a^4} \right) \cos ax + \left( \frac{x^3}{a} - \frac{6x}{a^3} \right) \operatorname{sen} ax$$

$$374. \int \frac{\cos ax}{x} dx = \ln x - \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} - \frac{(ax)^6}{6 \cdot 6!} + \dots = \ln x + \sum_{n=1}^{\infty} \frac{(-1)^n (ax)^{2n}}{(2n) \cdot (2n)!}$$

$$375. \int \frac{\cos ax}{x^2} dx = -\frac{\cos ax}{x} - a \int \frac{\operatorname{sen} ax}{x} dx \quad \text{Véase 374}$$

$$376. \int \frac{dx}{\cos ax} = \frac{1}{a} \ln(\sec ax + \operatorname{tg} ax) = \frac{1}{a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$377. \int \frac{x dx}{\cos ax} = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \dots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\} \quad E_n \text{ es } n^\circ \text{ de Euler}$$

$$378. \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\operatorname{sen} 2ax}{4a}$$

$$379. \int x \cos^2 ax \, dx = \frac{x^2}{4} + \frac{x \operatorname{sen} 2ax}{4a} + \frac{\cos 2ax}{8a^2}$$

$$380. \int \cos^3 ax \, dx = \frac{\operatorname{sen} ax}{a} - \frac{\operatorname{sen}^3 ax}{3a}$$

$$381. \int \cos^4 ax \, dx = \frac{3x}{8} + \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a}$$

$$382. \int \frac{dx}{\cos^2 ax} = \frac{1}{a} \operatorname{tg} ax$$

$$383. \int \frac{dx}{\cos^3 ax} = \frac{\operatorname{sen} ax}{2a \cos^2 ax} + \frac{1}{2a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

384.  $\int \cos px \cos qx \, dx = \frac{\operatorname{sen}(p-q)x}{2(p-q)} + \frac{\operatorname{sen}(p+q)x}{2(p+q)}$
385.  $\int \frac{dx}{1-\cos ax} = \frac{1}{a} \operatorname{cotg} \frac{ax}{2}$
386.  $\int \frac{x dx}{1-\cos ax} = -\frac{x}{a} \operatorname{cotg} \frac{ax}{2} + \frac{2}{a^2} \ln \operatorname{sen} \frac{ax}{2}$
387.  $\int \frac{dx}{1+\cos ax} = \frac{1}{a} \operatorname{tg} \frac{ax}{2}$
388.  $\int \frac{x dx}{1+\cos ax} = \frac{x}{a} \operatorname{tg} \frac{ax}{2} + \frac{2}{a^2} \ln \cos \frac{ax}{2}$
389.  $\int \frac{dx}{(1-\cos ax)^2} = -\frac{1}{2a} \operatorname{cotg} \frac{ax}{2} - \frac{1}{6a} \operatorname{cotg}^3 \frac{ax}{2}$
390.  $\int \frac{dx}{(1+\cos ax)^2} = \frac{1}{2a} \operatorname{tg} \frac{ax}{2} + \frac{1}{6a} \operatorname{tg}^3 \frac{ax}{2}$
391.  $\int \frac{dx}{p+q \cos ax} = \begin{cases} \frac{2}{a\sqrt{p^2-q^2}} \operatorname{tg}^{-1} \sqrt{\frac{p-q}{p+q}} \cdot \operatorname{tg} \frac{ax}{2} & \text{Si } p = \pm q, \\ \frac{1}{a\sqrt{q^2-p^2}} \ln \left( \frac{\operatorname{tg} \frac{ax}{2} + \sqrt{\frac{q+p}{q-p}}}{\operatorname{tg} \frac{ax}{2} - \sqrt{\frac{q+p}{q-p}}} \right) & \text{Véanse 385 y 387} \end{cases}$
392.  $\int \frac{dx}{(p+q \cos ax)^2} = \frac{\operatorname{sen} ax}{a(q^2-p^2)(p+q \cos ax)} - \frac{p}{(q^2-p^2)} \int \frac{dx}{p+q \cos ax}$  Si  $p = \pm q$ , véanse 389 y 390
393.  $\int \frac{dx}{p^2+q^2 \cos^2 ax} = \frac{1}{ap\sqrt{p^2+q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tg} ax}{\sqrt{p^2+q^2}}$
394.  $\int \frac{dx}{p^2-q^2 \cos^2 ax} = \begin{cases} \frac{1}{ap\sqrt{p^2+q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tg} ax}{\sqrt{p^2+q^2}} \\ \frac{1}{2ap\sqrt{q^2-p^2}} \ln \left( \frac{p \operatorname{tg} ax - \sqrt{q^2-p^2}}{p \operatorname{tg} ax + \sqrt{q^2-p^2}} \right) \end{cases}$
395.  $\int x^m \cos ax \, dx = \frac{x^m \operatorname{sen} ax}{a} + \frac{mx^{m-1} \cos ax}{a^2} - \frac{m(m-1)}{a^2} \int x^{m-2} \cos ax \, dx$
396.  $\int \frac{\cos ax}{x^n} \, dx = -\frac{\cos ax}{(n-1)x^{n-1}} - \frac{a}{n-1} \int \frac{\operatorname{sen} ax}{x^{n-1}} \, dx$  Véase 366
397.  $\int \cos^n ax \, dx = \frac{\cos^{n-1} ax \operatorname{sen} ax}{an} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$
398.  $\int \frac{dx}{\cos^n ax} = \frac{\operatorname{sen} ax}{a(n-1) \cos^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\cos^{n-2} ax}$
399.  $\int \frac{x dx}{\cos^n ax} = \frac{x \operatorname{sen} ax}{a(n-1) \cos^{n-1} ax} - \frac{1}{a^2(n-1)(n-2) \cos^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x dx}{\cos^{n-2} ax}$

INTEGRALES CON  $\operatorname{sen} ax$  y  $\cos ax$

400.  $\int \operatorname{sen} ax \cos ax \, dx = \frac{\operatorname{sen}^2 ax}{2a}$
401.  $\int \operatorname{sen} px \cos qx \, dx = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(p+q)x}{2(p+q)}$
402.  $\int \operatorname{sen}^n ax \cos ax \, dx = \frac{\operatorname{sen}^{n+1} ax}{(n+1)a}$  Si  $n = -1$ , véase 441
403.  $\int \operatorname{sen} ax \cos^n ax \, dx = -\frac{\cos^{n-1} ax}{(n+1)a}$  Si  $n = -1$ , véase 430
404.  $\int \operatorname{sen}^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\operatorname{sen} 4ax}{32a}$
405.  $\int \frac{dx}{\operatorname{sen} ax \cos ax} = \frac{1}{a} \ln \operatorname{tg} ax$

$$406. \int \frac{dx}{\operatorname{sen}^2 ax \cos ax} = \frac{1}{a} \ln \operatorname{tg} \left[ \frac{\pi}{4} + \frac{ax}{2} \right] - \frac{1}{a \operatorname{sen} ax}$$

$$407. \int \frac{dx}{\operatorname{sen} ax \cos^2 ax} = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + \frac{1}{a \cos ax}$$

$$408. \int \frac{dx}{\operatorname{sen}^2 ax \cos^2 ax} = -\frac{2 \cot g 2 ax}{a}$$

$$409. \int \frac{\operatorname{sen}^2 ax}{\cos ax} dx = \frac{1}{a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right) - \frac{\operatorname{sen} ax}{a}$$

$$410. \int \frac{\cos^2 ax}{\operatorname{sen} ax} dx = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + \frac{\cos ax}{a}$$

$$411. \int \frac{dx}{(1 \pm \operatorname{sen} ax) \cos ax} = \pm \frac{1}{2a(1 \pm \operatorname{sen} ax)} + \frac{1}{2a} \ln \operatorname{tg} \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

$$412. \int \frac{dx}{(1 \pm \cos ax) \operatorname{sen} ax} = \pm \frac{1}{2a(1 \pm \cos ax)} + \frac{1}{2a} \ln \operatorname{tg} \frac{ax}{2}$$

$$413. \int \frac{dx}{\operatorname{sen} ax \pm \cos ax} = -\frac{1}{a\sqrt{2}} \ln \operatorname{tg} \left( \pm \frac{\pi}{8} + \frac{ax}{2} \right)$$

$$414. \int \frac{\operatorname{sen} ax dx}{\operatorname{sen} ax \pm \cos ax} = \frac{x}{2} \mp \frac{1}{2a} \ln(\operatorname{sen} ax \pm \cos ax)$$

$$415. \int \frac{\cos ax dx}{\operatorname{sen} ax \pm \cos ax} = \pm \frac{x}{2} + \frac{1}{2a} \ln(\operatorname{sen} ax \pm \cos ax)$$

$$416. \int \frac{\operatorname{sen} ax dx}{p+q \cos ax} = -\frac{1}{aq} \ln(p+q \cos ax)$$

$$417. \int \frac{\cos ax dx}{p+q \operatorname{sen} ax} = \frac{1}{aq} \ln(p+q \operatorname{sen} ax)$$

$$418. \int \frac{\operatorname{sen} ax dx}{(p+q \cos ax)^n} = \frac{1}{aq(n-1)(p+q \cos ax)^{n-1}}$$

$$419. \int \frac{\cos ax dx}{(p+q \operatorname{sen} ax)^n} = \frac{-1}{aq(n-1)(p+q \operatorname{sen} ax)^{n-1}}$$

$$420. \int \frac{dx}{p \operatorname{sen} ax + q \cos ax} = \frac{1}{a\sqrt{p^2+q^2}} \ln \operatorname{tg} \left( \frac{ax + \operatorname{tg}}{2} \right)$$

$$421. \int \frac{dx}{p \operatorname{sen} ax + q \cos ax + r} = \begin{cases} \frac{2}{a\sqrt{r^2-p^2-q^2}} \operatorname{tg}^{-1} \left( \frac{p+(r-q) \operatorname{tg}(ax/2)}{\sqrt{r^2-p^2-q^2}} \right) \\ \frac{1}{a\sqrt{p^2+q^2-r^2}} \ln \left( \frac{p-\sqrt{p^2+q^2-r^2}+(r-q) \operatorname{tg}(ax/2)}{p+\sqrt{p^2+q^2-r^2}+(r-q) \operatorname{tg}(ax/2)} \right) \end{cases}$$

Si  $r = q$  véase 422. Si  $r^2 = p^2$  véase 423

$$422. \int \frac{dx}{p \operatorname{sen} ax + q(1 + \cos ax)} = \frac{1}{ap} \ln(q + p \operatorname{tg} \frac{ax}{2})$$

$$423. \int \frac{dx}{p \operatorname{sen} ax + q \cos ax \pm \sqrt{p^2+q^2}} = \frac{-1}{a\sqrt{p^2+q^2}} \operatorname{tg} \left( \frac{\pi}{4} \mp \frac{ax + \operatorname{tg}^{-1} q/p}{2} \right)$$

$$424. \int \frac{dx}{p^2 \operatorname{sen}^2 ax + q^2 \cos^2 ax} = \frac{1}{apq} \operatorname{tg}^{-1} \left( \frac{p \operatorname{tg} ax}{q} \right)$$

$$425. \int \frac{dx}{p^2 \operatorname{sen}^2 ax - q^2 \cos^2 ax} = \frac{1}{2apq} \ln \left( \frac{p \operatorname{tg} ax - q}{p \operatorname{tg} ax + q} \right)$$

$$426. \int \operatorname{sen}^m ax \cos^n ax dx = \begin{cases} -\frac{\operatorname{sen}^{m-1} ax \cos^{n-1} ax}{a(m+n)} + \frac{(m-1)}{(m+n)} \int \operatorname{sen}^{m-2} ax \cos^n ax dx \\ \frac{\operatorname{sen}^{m+1} ax \cos^{n-1} ax}{a(m+n)} + \frac{(n-1)}{(m+n)} \int \operatorname{sen}^m ax \cos^{n-2} ax dx \end{cases}$$

$$427. \int \frac{\operatorname{sen}^m ax}{\cos^n ax} dx = \begin{cases} \frac{\operatorname{sen}^{m-1} ax}{a(n-1) \cos^{n-1} ax} + \frac{m-1}{n-1} \int \frac{\operatorname{sen}^{m-2} ax}{\cos^{n-1} ax} dx \\ \frac{\operatorname{sen}^{m+1} ax}{a(n-1) \cos^{n-1} ax} - \frac{m-1}{n-1} \int \frac{\operatorname{sen}^m ax}{\cos^{n-2} ax} dx \\ \frac{-\operatorname{sen}^{m-1} ax}{a(m-n) \cos^{n-1} ax} + \frac{m-1}{n-1} \int \frac{\operatorname{sen}^{m-2} ax}{\cos^n ax} dx \end{cases}$$

$$428. \int \frac{\cos^m ax}{\operatorname{sen}^n ax} dx = \begin{cases} \frac{-\cos^{m-1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{m-1}{n-1} \int \frac{\cos^{m-2} ax}{\operatorname{sen}^{n-2} ax} dx \\ \frac{-\cos^{m+1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{m-n+2}{n-1} \int \frac{\cos^m ax}{\operatorname{sen}^{n-2} ax} dx \\ \frac{\cos^{m-1} ax}{a(m-n)\operatorname{sen}^{n-1} ax} + \frac{m-1}{n-1} \int \frac{\cos^{m-2} ax}{\operatorname{sen}^n ax} dx \end{cases}$$

$$429. \int \operatorname{sen}^m ax \cos^n ax dx = \begin{cases} \frac{1}{a(n-1)\operatorname{sen}^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{n-1} \int \frac{dx}{\operatorname{sen}^m ax \cos^{n-2} ax} \\ \frac{-1}{a(m-1)\operatorname{sen}^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{m-1} \int \frac{dx}{\operatorname{sen}^{m-2} ax \cos^n ax} \end{cases}$$

### INTEGRALES CON $\operatorname{tg} ax$

$$430. \int \operatorname{tg} ax dx = -\frac{1}{a} \ln \cos ax = \frac{1}{a} \ln \sec ax$$

$$431. \int \operatorname{tg}^2 ax dx = \frac{\operatorname{tg} ax}{a} - x$$

$$432. \int \operatorname{tg}^3 ax dx = \frac{\operatorname{tg}^2 ax}{2a} + \frac{1}{a} \ln \cos ax$$

$$433. \int \operatorname{tg}^n ax \sec^2 ax dx = \frac{\operatorname{tg}^{n+1} ax}{(n+1)a}$$

$$434. \int \frac{\sec^2 ax}{\operatorname{tg} ax} dx = \frac{1}{a} \ln \operatorname{tg} ax$$

$$435. \int \frac{dx}{\operatorname{tg} ax} = \frac{1}{a} \ln \operatorname{sen} ax$$

$$436. \int x \operatorname{tg} ax dx = \frac{1}{a^2} \left\{ \frac{(ax)^3}{3} + \frac{(ax)^5}{5} + \frac{2(ax)^7}{105} + \dots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \dots \right\}$$

$B_n$  es  $n^\circ$  de Bernoulli tanto en 436 como en 437.

$$437. \int \frac{\operatorname{tg} ax}{x} dx = ax + \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} + \dots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \dots$$

$$438. \int x \operatorname{tg}^2 ax dx = \frac{x \operatorname{tg} ax}{a} + \frac{1}{a^2} \ln \cos ax - \frac{x^2}{2}$$

$$439. \int \frac{dx}{p+q \operatorname{tg} ax} = \frac{px}{p^2+q^2} + \frac{q}{a(p^2+q^2)} \ln(q \operatorname{sen} ax + p \cos ax)$$

$$440. \int \operatorname{tg}^n ax dx = \frac{\operatorname{tg}^{n-1} ax}{(n-1)a} - \int \operatorname{tg}^{n-2} ax dx$$

### INTEGRALES CON $\operatorname{cotg} ax$

$$441. \int \operatorname{cotg} ax dx = \frac{1}{a} \ln \operatorname{sen} ax$$

$$442. \int \operatorname{cotg}^2 ax dx = -\frac{\operatorname{cotg} ax}{a} - x$$

$$443. \int \operatorname{cotg}^3 ax dx = -\frac{\operatorname{cotg}^2 ax}{2a} - \frac{1}{a} \ln \operatorname{sen} ax$$

$$444. \int \operatorname{cotg}^n ax \operatorname{cosec}^2 ax dx = -\frac{\operatorname{cotg}^{n+1} ax}{(n+1)a}$$

$$445. \int \frac{\operatorname{cosec}^2 ax}{\operatorname{cotg} ax} dx = -\frac{1}{a} \ln \operatorname{cotg} ax$$

$$446. \int \frac{dx}{\operatorname{cotg} ax} = -\frac{1}{a} \ln \cos ax$$

$$447. \int x \operatorname{cotg} ax dx = \frac{1}{a^2} \left( ax - \frac{(ax)^3}{9} - \frac{(ax)^5}{225} - \dots - \frac{2^{2n} B_n(ax)^{2n+1}}{(2n+1)!} \right)$$

$B_n$  es  $n^\circ$  de Bernoulli tanto en 447 como en 448

$$448. \int \frac{\operatorname{cotg} ax}{x} dx = -\frac{1}{ax} - \frac{ax}{3} - \frac{(ax)^3}{135} - \dots - \frac{2^{2n} B_n(ax)^{2n-1}}{(2n-1)(2n)!}$$

$$449. \int x \cot g^2 ax \, dx = \frac{x \cot g ax}{a} + \frac{1}{a^2} \ln \operatorname{sen} ax - \frac{x^2}{2}$$

$$450. \int \frac{dx}{p+q \cot g ax} = \frac{px}{p^2+q^2} - \frac{q}{a(p^2+q^2)} \ln(q \operatorname{sen} ax + p \cos ax)$$

$$451. \int \cot g^n ax \, dx = -\frac{\cot g^{n-1} ax}{(n-1)a} - \int \cot g^{n-2} ax \, dx$$

INTEGRALES CON sec ax

$$452. \int \sec ax \, dx = \frac{1}{a} \ln(\sec ax + \operatorname{tg} ax) = \frac{1}{a} \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$$

$$453. \int \sec^2 ax \, dx = \frac{\operatorname{tg} ax}{a}$$

$$454. \int \sec^3 ax \, dx = \frac{\sec ax \operatorname{tg} ax}{2a} + \frac{1}{2a} \ln(\sec ax + \operatorname{tg} ax)$$

$$455. \int \sec^n ax \operatorname{tg} ax \, dx = \frac{\sec^{n-1} ax}{n a}$$

$$456. \int \frac{dx}{\sec ax} = \frac{\operatorname{sen} ax}{a}$$

$$457. \int x \sec ax \, dx = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \dots + \frac{E_n(ax)^{2n+1}}{(2n+2)(2n)!} \right\}$$

$E_n$  es  $n^\circ$  de Euler

$$458. \int \frac{\sec ax}{x} \, dx = \ln x + \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} + \frac{61(ax)^6}{4320} + \dots + \frac{E_n(ax)^{2n}}{2n(2n)!}$$

$$459. \int x \sec^2 ax \, dx = \frac{x}{a} \operatorname{tg} ax + \frac{1}{a^2} \ln \cos ax$$

$$460. \int \frac{dx}{q+p \sec ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{q+p \cos ax} \quad \text{Véase 391}$$

$$461. \int \sec^n ax \, dx = \frac{\sec^{n-2} ax \operatorname{tg} ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx$$

INTEGRALES CON cosec ax

$$462. \int \operatorname{cosec} ax \, dx = \frac{1}{a} \ln(\operatorname{cosec} ax - \cot g ax) = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2}$$

$$463. \int \operatorname{cosec}^2 ax \, dx = -\frac{\cot g ax}{a}$$

$$464. \int \operatorname{cosec}^3 ax \, dx = -\frac{\operatorname{cosec} ax \cot g ax}{2a} + \frac{1}{2a} \ln \operatorname{tg} \frac{ax}{2}$$

$$465. \int \operatorname{cosec}^n ax \cot g ax \, dx = -\frac{\operatorname{cosec}^{n-1} ax}{n a}$$

$$466. \int \frac{dx}{\operatorname{cosec} ax} = -\frac{\cos ax}{a}$$

$$467. \int x \operatorname{cosec} ax \, dx = \frac{1}{a^2} \left\{ ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \dots + \frac{2(2^{2n-1}-1)E_n(ax)^{2n+1}}{(2n+1)!} + \dots \right\}$$

$B_n$  es  $n^\circ$  de Bernoulli

$$468. \int \frac{\operatorname{cosec} ax}{x} \, dx = -\frac{1}{ax} + \frac{ax}{6} + \frac{7(ax)^3}{1080} + \dots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \dots$$

$$469. \int x \operatorname{cosec}^2 ax \, dx = -\frac{x}{a} \cot g ax + \frac{1}{a^2} \ln \operatorname{sen} ax$$

$$470. \int \frac{dx}{q+p \operatorname{cosec} ax} = \frac{x}{p} - \frac{q}{p} \int \frac{dx}{q+p \operatorname{sen} ax} \quad \text{Véase 361}$$

$$471. \int \operatorname{cosec} ax \, dx = -\frac{\operatorname{cosec}^{n-2} ax \cot g ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} ax \, dx$$

INTEGRALES DE FUNCIONES TRIGONOMETRICAS INVERSAS

24 472.  $\int \operatorname{sen}^{-1} \frac{x}{a} \, dx = x \operatorname{sen}^{-1} \frac{x}{a} + \sqrt{a^2 - x^2}$

473.  $\int x \operatorname{sen}^{-1} \frac{x}{a} dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \operatorname{sen}^{-1} \frac{x}{a} + \frac{x\sqrt{a^2-x^2}}{4}$
474.  $\int x^2 \operatorname{sen}^{-1} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{sen}^{-1} \frac{x}{a} + \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$
475.  $\int x^m \operatorname{sen}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{sen}^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{a^2-x^2}} dx$
476.  $\int \frac{\operatorname{sen}^{-1}(x/a)}{x} dx = \frac{x}{a} + \frac{(x/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 \cdot (x/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \cdot (x/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots$
477.  $\int \frac{\operatorname{sen}^{-1}(x/a)}{x^2} dx = -\frac{\operatorname{sen}^{-1}(x/a)}{x} - \frac{1}{a} \ln \left( \frac{a + \sqrt{a^2-x^2}}{x} \right)$
478.  $\int (\operatorname{sen}^{-1} \frac{x}{a})^2 dx = x (\operatorname{sen}^{-1} \frac{x}{a})^2 - 2x + 2\sqrt{a^2-x^2} \operatorname{sen}^{-1} \frac{x}{a}$
479.  $\int \cos^{-1} \frac{x}{a} dx = x \cos^{-1} \frac{x}{a} - \sqrt{a^2-x^2}$
480.  $\int x \cos^{-1} \frac{x}{a} dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \cos^{-1} \frac{x}{a} - \frac{\sqrt{a^2-x^2}}{4}$
481.  $\int x^2 \cos^{-1} \frac{x}{a} dx = \frac{x^3}{3} \cos^{-1} \frac{x}{a} - \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$
482.  $\int x^m \cos^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \cos^{-1} \frac{x}{a} + \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{a^2-x^2}} dx$
483.  $\int \frac{\cos^{-1}(x/a)}{x} dx = \frac{\pi}{2} \ln x - \int \frac{\operatorname{sen}^{-1}(x/a)}{x} dx$  Véase 476
484.  $\int \frac{\cos^{-1}(x/a)}{x^2} dx = -\frac{\cos^{-1}(x/a)}{x} + \frac{1}{a} \ln \left( \frac{a + \sqrt{a^2-x^2}}{x} \right)$
485.  $\int (\cos^{-1} \frac{x}{a})^2 dx = x (\cos^{-1} \frac{x}{a})^2 - 2x - 2\sqrt{a^2-x^2} \cos^{-1} \frac{x}{a}$
486.  $\int \operatorname{tg}^{-1} \frac{x}{a} dx = x \operatorname{tg}^{-1} \frac{x}{a} - \frac{a}{2} \ln(x^2 + a^2)$
487.  $\int x \operatorname{tg}^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 + a^2) \operatorname{tg}^{-1} \frac{x}{a} - \frac{ax}{2}$
488.  $\int x^2 \operatorname{tg}^{-1} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{tg}^{-1} \frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6} \ln(x^2 + a^2)$
489.  $\int x^m \operatorname{tg}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{tg}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{x^2+a^2} dx$
490.  $\int \frac{\operatorname{tg}^{-1}(x/a)}{x} dx = \frac{x}{a} - \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} - \frac{(x/a)^7}{7^2} + \dots$
491.  $\int \frac{\operatorname{tg}^{-1}(x/a)}{x^2} dx = -\frac{1}{x} \operatorname{tg}^{-1} \frac{x}{a} - \frac{1}{2a} \ln \left( \frac{x^2+a^2}{x^2} \right)$
492.  $\int \operatorname{cotg}^{-1} \frac{x}{a} dx = x \operatorname{cotg}^{-1} \frac{x}{a} + \frac{a}{2} \ln(x^2 + a^2)$
493.  $\int x \operatorname{cotg}^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 + a^2) \operatorname{cotg}^{-1} \frac{x}{a} + \frac{ax}{2}$
494.  $\int x^2 \operatorname{cotg}^{-1} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{cotg}^{-1} \frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6} \ln(x^2 + a^2)$
495.  $\int x^m \operatorname{cotg}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{cotg}^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^{m+1}}{x^2+a^2} dx$
496.  $\int \frac{\operatorname{cotg}^{-1}(x/a)}{x} dx = \frac{\pi}{2} \ln x - \int \frac{\operatorname{tg}^{-1}(x/a)}{x} dx$  Véase 490
497.  $\int \frac{\operatorname{cotg}^{-1}(x/a)}{x^2} dx = -\frac{1}{x} \operatorname{cotg}^{-1} \frac{x}{a} + \frac{1}{2a} \ln \left( \frac{x^2+a^2}{x^2} \right)$
498.  $\int \sec^{-1} \frac{x}{a} dx = \begin{cases} x \sec^{-1} \frac{x}{a} - a \ln(x + \sqrt{x^2 - a^2}); & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ x \sec^{-1} \frac{x}{a} + a \ln(x + \sqrt{x^2 - a^2}); & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$
499.  $\int x \sec^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^2}{2} \sec^{-1} \frac{x}{a} - \frac{a\sqrt{x^2-a^2}}{2}; & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^2}{2} \sec^{-1} \frac{x}{a} + \frac{a\sqrt{x^2-a^2}}{2}; & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$

$$500. \int x^2 \sec^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^3}{3} \sec^{-1} \frac{x}{a} - \frac{\alpha \sqrt{x^2 - a^2}}{2} - \frac{a^3}{6} \ln(x + \sqrt{x^2 - a^2}); & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^3}{3} \sec^{-1} \frac{x}{a} + \frac{\alpha \sqrt{x^2 - a^2}}{2} + \frac{a^3}{6} \ln(x + \sqrt{x^2 - a^2}); & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$501. \int x^m \sec^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \sec^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 - a^2}}; & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^{m+1}}{m+1} \sec^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 - a^2}}; & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$502. \int \frac{\sec^{-1}(x/a)}{x} dx = \frac{\pi}{2} \ln x + \frac{a}{x} + \frac{(x/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 \cdot (x/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \cdot (x/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots$$

$$503. \int \frac{\sec^{-1}(x/a)}{x^2} dx = \begin{cases} -\frac{\sec^{-1}(x/a)}{x} + \frac{\sqrt{x^2 - a^2}}{\alpha x}; & 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2} \\ -\frac{\sec^{-1}(x/a)}{x} - \frac{\sqrt{x^2 - a^2}}{\alpha x}; & \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi \end{cases}$$

$$504. \int \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} x \operatorname{cosec}^{-1} \frac{x}{a} + a \ln(x + \sqrt{x^2 - a^2}); & 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ x \operatorname{cosec}^{-1} \frac{x}{a} - a \ln(x + \sqrt{x^2 - a^2}); & -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$505. \int x \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^2}{2} \operatorname{cosec}^{-1} \frac{x}{a} + \frac{a \sqrt{x^2 - a^2}}{2}; & 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^2}{2} \operatorname{cosec}^{-1} \frac{x}{a} - \frac{a \sqrt{x^2 - a^2}}{2}; & -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$506. \int x^2 \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^3}{3} \operatorname{cosec}^{-1} \frac{x}{a} + \frac{\alpha \sqrt{x^2 - a^2}}{2} + \frac{a}{6} \ln(x + \sqrt{x^2 - a^2}); & 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^3}{3} \operatorname{cosec}^{-1} \frac{x}{a} - \frac{\alpha \sqrt{x^2 - a^2}}{2} - \frac{a}{6} \ln(x + \sqrt{x^2 - a^2}); & -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

$$507. \int x^m \operatorname{cosec}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \operatorname{cosec}^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 - a^2}}; & 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ \frac{x^{m+1}}{m+1} \operatorname{cosec}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 - a^2}}; & \frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < \pi \end{cases}$$

$$508. \int \frac{\operatorname{cosec}^{-1}(x/a)}{x} dx = -\left\{ \frac{a}{x} + \frac{(x/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 \cdot (x/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \cdot (x/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots \right.$$

$$509. \int \frac{\operatorname{cosec}^{-1}(x/a)}{x^2} dx = \begin{cases} -\frac{\operatorname{cosec}^{-1}(x/a)}{x} - \frac{\sqrt{x^2 - a^2}}{\alpha x}; & 0 < \operatorname{cosec}^{-1} \frac{x}{a} < \frac{\pi}{2} \\ -\frac{\operatorname{cosec}^{-1}(x/a)}{x} + \frac{\sqrt{x^2 - a^2}}{\alpha x}; & -\frac{\pi}{2} < \operatorname{cosec}^{-1} \frac{x}{a} < 0 \end{cases}$$

INTEGRALES CON  $e^{ax}$

$$510. \int e^{ax} dx = \frac{e^{ax}}{a}$$

$$511. \int x e^{ax} dx = \frac{e^{ax}}{a} \left( x - \frac{1}{a} \right)$$

$$512. \int x^2 e^{ax} dx = \frac{e^{ax}}{a} \left( x^2 - \frac{2x}{a} + \frac{2}{a^2} \right)$$

$$513. \int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \quad \left( = \sum_{k=1}^n \frac{(-1)^k k!}{a^k} \quad \text{Si } n \text{ es natural} \right)$$

$$514. \int \frac{e^{ax}}{x} dx = \ln x + \frac{\alpha}{1 \cdot 1!} + \frac{(\alpha)^2}{2 \cdot 2!} + \frac{(\alpha)^3}{3 \cdot 3!} + \dots$$

$$515. \int \frac{e^{ax}}{x^n} dx = \frac{-e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{e^{ax}}{x^{n-1}} dx$$

$$516. \int \frac{dx}{p+q e^{ax}} = \frac{x}{p} - \frac{1}{ap} \ln(p+q e^{ax})$$

$$517. \int \frac{dx}{(p+q e^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p+q e^{ax})} - \frac{1}{ap^2} \ln(p+q e^{ax})$$

$$518. \int \frac{dx}{p e^{ax} + q e^{-ax}} = \begin{cases} \frac{1}{a \sqrt{pq}} \operatorname{tg}^{-1} \left( \sqrt{\frac{p}{q}} e^{ax} \right) \\ \frac{1}{2a \sqrt{-pq}} \ln \left( \frac{e^{ax} - \sqrt{-q/p}}{e^{ax} + \sqrt{-q/p}} \right) \end{cases}$$

$$519. \int e^{ax} \operatorname{sen} bx dx = \frac{e^{ax} (a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2}$$

$$520. \int e^{ax} \cos bx dx = \frac{e^{ax} (a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2}$$

$$521. \int x e^{ax} \operatorname{sen} bx dx = \frac{x e^{ax} (a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2} - \frac{e^{ax} ((a^2 - b^2) \operatorname{sen} bx - 2ab \cos bx)}{(a^2 + b^2)^2}$$

$$522. \int x e^{ax} \cos bx dx = \frac{x e^{ax} (a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2} - \frac{e^{ax} ((a^2 - b^2) \cos bx + 2ab \operatorname{sen} bx)}{(a^2 + b^2)^2}$$

$$523. \int e^{ax} \ln x dx = \frac{e^{ax} \ln x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$$

$$524. \int e^{ax} \operatorname{sen}^n bx dx = \frac{e^{ax} \operatorname{sen}^{-1} bx (a \operatorname{sen} bx - nb \cos bx)}{a^2 + n^2 b^2} + \frac{n(n+1)b^2}{a^2 + n^2 b^2} \int e^{ax} \operatorname{sen}^{n-2} bx dx$$

$$525. \int e^{ax} \cos^n bx dx = \frac{e^{ax} \cos^{-1} bx (a \cos bx + nb \operatorname{sen} bx)}{a^2 + n^2 b^2} + \frac{n(n-1)b^2}{a^2 + n^2 b^2} \int e^{ax} \cos^{n-2} bx dx$$

### INTEGRALES CON $\ln x$

$$526. \int \ln x dx = x \ln x - x$$

$$527. \int x \ln x dx = \frac{x^2}{2} (\ln x - \frac{1}{2})$$

$$528. \int x^m \ln x dx = \frac{x^{m+1}}{m+1} (\ln x - \frac{1}{m+1}) \quad \text{Si } m = -1, \text{ véase } 529$$

$$529. \int \frac{\ln x}{x} dx = \frac{1}{2} \ln^2 x$$

$$530. \int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x}$$

$$531. \int \ln^2 x dx = x \ln^2 x - 2x \ln x + 2x$$

$$532. \int \frac{\ln^n x}{x} dx = \frac{\ln^{n+1} x}{n+1} \quad \text{Si } n = -1, \text{ véase } 533$$

$$533. \int \frac{dx}{x \ln x} = \ln(\ln x)$$

$$534. \int \frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{\ln^2 x}{2 \cdot 2!} + \frac{\ln^3 x}{3 \cdot 3!} + \dots$$

$$535. \int \frac{x^m dx}{\ln x} = \ln(\ln x) + (m+1) \ln x + \frac{(m+1)^2 \ln^2 x}{2 \cdot 2!} + \frac{(m+1)^3 \ln^3 x}{3 \cdot 3!} + \dots$$

$$536. \int \ln^n x dx = x \ln^n x - n \int \ln^{n-1} x dx$$

$$537. \int x^m \ln^n x dx = \frac{\ln^n x}{m+1} - \frac{n}{m+1} \int x^m \ln^{n-1} x dx \quad \text{Si } m = -1, \text{ véase } 532$$

$$538. \int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) - 2x + 2a \operatorname{tg}^{-1} \frac{x}{a}$$

$$539. \int \ln(x^2 - a^2) dx = x \ln(x^2 - a^2) - 2x + a \ln \left( \frac{x+a}{x-a} \right)$$

$$540. \int x^m \ln(x^2 \pm a^2) dx = \frac{x^{m+1} \ln(x^2 \pm a^2)}{m+1} - \frac{2}{m+1} \int \frac{x^{m+2}}{(x^2 \pm a^2)} dx$$

### INTEGRALES CON $\operatorname{senh} ax$

$$541. \int \operatorname{senh} ax dx = \frac{\operatorname{cosh} ax}{a}$$

$$542. \int x \operatorname{senh} ax dx = -\frac{\operatorname{senh} ax}{a^2} + \frac{x \operatorname{cosh} ax}{a}$$

$$543. \int x^2 \sinh ax \, dx = -\frac{2x}{a^2} \sinh ax + \left(\frac{2}{a^3} + \frac{x^2}{a}\right) \cosh ax$$

$$544. \int \frac{\sinh ax}{x} \, dx = ax + \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \dots = \sum_{n=1}^{\infty} \frac{(ax)^{2n-1}}{(2n-1)!}$$

$$545. \int \frac{\sinh ax}{x^2} \, dx = -\frac{\sinh ax}{x} + a \int \frac{\cosh ax}{x} \, dx \quad \text{Véase 566}$$

$$546. \int \frac{dx}{\sinh ax} = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2}$$

$$547. \int \frac{x dx}{\sinh ax} = \frac{1}{a^2} \left\{ ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \dots + \frac{2(-1)^n (2^{2n-1}-1) B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}$$

$B_n$  es  $n^\circ$  de Bernoulli

$$548. \int \sinh^2 ax \, dx = -\frac{x}{2} + \frac{\sinh ax \cdot \cosh ax}{2a}$$

$$549. \int x \sinh^2 ax \, dx = -\frac{x^2}{4} + \frac{x \sinh 2ax}{4a} - \frac{\cosh 2ax}{8a^2}$$

$$550. \int \frac{dx}{\sinh^2 ax} = -\frac{1}{a} \operatorname{cotgh} ax$$

$$551. \int \sinh px \sinh qx \, dx = -\frac{\sinh(p-q)x}{2(p-q)} + \frac{\sinh(p+q)x}{2(p+q)} \quad \text{Si } p = \pm q, \text{ véase 548}$$

$$552. \int \sinh px \sin qx \, dx = \frac{p \cosh px \sin qx - q \sinh px \cos qx}{p^2 + q^2}$$

$$553. \int \sinh px \cos qx \, dx = \frac{p \cosh px \cos qx + q \sinh px \sin qx}{p^2 + q^2}$$

$$554. \int \frac{dx}{p+q \sinh ax} = \frac{1}{a \sqrt{p^2+q^2}} \ln \left( \frac{qe^{ax} + p - \sqrt{p^2+q^2}}{qe^{ax} + p + \sqrt{p^2+q^2}} \right)$$

$$555. \int \frac{dx}{(p+q \sinh ax)^2} = \frac{-q \cosh ax}{a(p^2+q^2)(p+q \sinh ax)} + \frac{p}{(p^2+q^2)} \int \frac{dx}{p+q \sinh ax}$$

$$556. \int \frac{dx}{p^2+q^2 \sinh^2 ax} = \begin{cases} \frac{1}{ap \sqrt{q^2-p^2}} \operatorname{tg}^{-1} \frac{\sqrt{q^2-p^2} \cdot \operatorname{tgh} ax}{p} \\ \frac{1}{2ap \sqrt{p^2-q^2}} \ln \left( \frac{p+\sqrt{p^2-q^2} \cdot \operatorname{tgh} ax}{p-\sqrt{p^2-q^2} \cdot \operatorname{tgh} ax} \right) \end{cases}$$

$$557. \int \frac{dx}{p^2-q^2 \sinh^2 ax} = \frac{1}{2ap \sqrt{p^2+q^2}} \ln \left( \frac{p+\sqrt{p^2+q^2} \cdot \operatorname{tgh} ax}{p-\sqrt{p^2+q^2} \cdot \operatorname{tgh} ax} \right)$$

$$558. \int x^m \sinh ax \, dx = \frac{x^m \cosh ax}{a} - \frac{m}{a} \int x^{m-1} \cosh ax \, dx \quad \text{Véase 586}$$

$$559. \int \frac{\sinh ax}{x^n} \, dx = -\frac{\sinh ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\cosh ax}{x^{n-1}} \, dx \quad \text{Véase 587}$$

$$560. \int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{an} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx$$

$$561. \int \frac{dx}{\sinh^n ax} = \frac{-\cosh ax}{a(n-1) \sinh^{n-1} ax} - \frac{n-2}{n-1} \int \frac{dx}{\sinh^{n-2} ax}$$

$$562. \int \frac{x dx}{\sinh^n ax} = \frac{-x \cosh ax}{a(n-1) \sinh^{n-1} ax} - \frac{1}{a^2(n-1)(n-2) \sinh^{n-3} ax} - \frac{n-2}{n-1} \int \frac{x dx}{\sinh^{n-2} ax}$$

INTEGRALES CON  $\cosh ax$

$$563. \int \cosh ax \, dx = \frac{\sinh ax}{a}$$

$$564. \int x \cosh ax \, dx = -\frac{\cosh ax}{a^2} + \frac{x \sinh ax}{a}$$

$$565. \int x^2 \cosh ax \, dx = -\frac{2x}{a^2} \cosh ax + \left(\frac{x^2}{a} + \frac{2}{a^3}\right) \sinh ax$$

$$566. \int \frac{\cosh ax}{x} \, dx = \ln x + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} + \frac{(ax)^6}{6 \cdot 6!} + \dots = \ln x + \sum_{n=1}^{\infty} \frac{(ax)^{2n}}{(2n) \cdot (2n)!}$$

567.  $\int \frac{\cosh ax}{x^2} dx = -\frac{\cosh ax}{x} + a \int \frac{\sinh ax}{x} dx$  Véase 544
568.  $\int \frac{dx}{\cosh ax} = \frac{2}{a} \operatorname{tg}^{-1} e^{ax}$
569.  $\int \frac{x dx}{\cosh ax} = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} - \dots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\}$   $E_n$  es  $n^\circ$  de Euler
570.  $\int \cosh^2 ax dx = \frac{x}{2} + \frac{\sinh ax \cosh ax}{2}$
571.  $\int x \cosh^2 ax dx = \frac{x^2}{4} + \frac{x \sinh 2ax}{4a} - \frac{\cosh 2ax}{8a^2}$
572.  $\int \frac{dx}{\cosh^2 ax} = \frac{1}{a} \operatorname{tgh} ax$
573.  $\int \cosh px \cosh qx dx = \frac{\sinh(p-q)x}{2(p-q)} + \frac{\sinh(p+q)x}{2(p+q)}$  Si  $p = \pm q$ , véase 570
574.  $\int \cosh px \operatorname{sen} qx dx = \frac{p \sinh px \operatorname{sen} qx - q \cosh px \cos qx}{p^2 + q^2}$
575.  $\int \cosh px \cos qx dx = \frac{p \sinh px \cos qx + q \cosh px \operatorname{sen} qx}{p^2 + q^2}$
576.  $\int \frac{dx}{1 - \cosh ax} = \frac{1}{a} \cot \operatorname{gh} \frac{ax}{2}$
577.  $\int \frac{x dx}{1 - \cosh ax} = \frac{x}{a} \cot \operatorname{gh} \frac{ax}{2} - \frac{2}{a^2} \ln \sinh \frac{ax}{2}$
578.  $\int \frac{dx}{1 + \cosh ax} = \frac{1}{a} \operatorname{tgh} \frac{ax}{2}$
579.  $\int \frac{x dx}{1 + \cosh ax} = \frac{x}{a} \operatorname{tgh} \frac{ax}{2} - \frac{2}{a^2} \ln \cosh \frac{ax}{2}$
580.  $\int \frac{dx}{(1 - \cosh ax)^2} = \frac{1}{2a} \cot \operatorname{gh} \frac{ax}{2} - \frac{1}{6a} \cot \operatorname{gh}^3 \frac{ax}{2}$
581.  $\int \frac{dx}{(1 + \cosh ax)^2} = \frac{1}{2a} \operatorname{tgh} \frac{ax}{2} - \frac{1}{6a} \operatorname{tgh}^3 \frac{ax}{2}$
582.  $\int \frac{dx}{p + q \cosh ax} = \begin{cases} \frac{2}{a \sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \frac{p + qe^{ax}}{\sqrt{q^2 - p^2}} \\ \frac{1}{a \sqrt{p^2 - q^2}} \ln \left( \frac{qe^{ax} + p - \sqrt{p^2 - q^2}}{qe^{ax} + p + \sqrt{p^2 - q^2}} \right) \end{cases}$
583.  $\int \frac{dx}{(p + q \cosh ax)^2} = \frac{q \sinh ax}{a(q^2 - p^2)(p + q \cosh ax)} - \frac{p}{(q^2 - p^2)} \int \frac{dx}{p + q \cosh ax}$
584.  $\int \frac{dx}{p^2 + q^2 \cosh^2 ax} = \begin{cases} \frac{1}{2ap \sqrt{p^2 + q^2}} \ln \left( \frac{p \operatorname{tgh} ax + \sqrt{p^2 + q^2}}{p \operatorname{tgh} ax - \sqrt{p^2 + q^2}} \right) \\ \frac{1}{ap \sqrt{p^2 + q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tgh} ax}{\sqrt{p^2 + q^2}} \end{cases}$
585.  $\int \frac{dx}{p^2 - q^2 \cosh^2 ax} = \begin{cases} \frac{-1}{ap \sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tgh} ax}{\sqrt{q^2 - p^2}} \\ \frac{1}{2ap \sqrt{p^2 - q^2}} \ln \left( \frac{p \operatorname{tgh} ax + \sqrt{p^2 - q^2}}{p \operatorname{tgh} ax - \sqrt{p^2 - q^2}} \right) \end{cases}$
586.  $\int x^m \cosh ax dx = \frac{x^m \sinh ax}{a} - \frac{m}{a} \int x^{m-1} \sinh ax dx$  Véase 558
587.  $\int \frac{\cosh ax}{x^n} dx = -\frac{\cosh ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\sinh ax}{x^{n-1}} dx$  Véase 559
588.  $\int \cosh^n ax dx = \frac{\cosh^{n-1} ax \sinh ax}{an} + \frac{n-1}{n} \int \cosh^{n-2} ax dx$
589.  $\int \frac{dx}{\cosh^n ax} = \frac{\sinh ax}{a(n-1) \cosh^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cosh^{n-2} ax}$
590.  $\int \frac{x dx}{\cosh^n ax} = \frac{x \sinh ax}{a(n-1) \cosh^{n-1} ax} + \frac{1}{a^2(n-1)(n-2) \cosh^{n-2} ax} + \frac{n-2}{n-1} \int \frac{x dx}{\cosh^{n-2} ax}$

591.  $\int \sinh ax \cosh ax \, dx = \frac{\sinh^2 ax}{2a}$
592.  $\int \sinh px \cosh qx \, dx = \frac{\cosh(p-q)x}{2(p-q)} + \frac{\cosh(p+q)x}{2(p+q)}$
593.  $\int \sinh^n ax \cosh ax \, dx = \frac{\sinh^{n+1} ax}{(n+1)a}$  Si  $n = -1$ , véase 615
594.  $\int \sinh ax \cosh^n ax \, dx = \frac{\cosh^{n+1} ax}{(n+1)a}$  Si  $n = -1$ , véase 604
595.  $\int \sinh^2 ax \cosh^2 ax \, dx = -\frac{x}{8} + \frac{\sinh 4ax}{32a}$
596.  $\int \frac{dx}{\sinh ax \cosh ax} = \frac{1}{a} \ln \operatorname{tgh} ax$
597.  $\int \frac{dx}{\sinh^2 ax \cosh ax} = -\frac{1}{a} \operatorname{tg}^{-1} \sinh ax - \frac{\cosh ax}{a}$
598.  $\int \frac{dx}{\sinh ax \cosh^2 ax} = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\sec h ax}{a}$
599.  $\int \frac{dx}{\sinh^2 ax \cosh^2 ax} = -\frac{2 \cot gh 2ax}{a}$
600.  $\int \frac{\sinh^2 ax}{\cosh ax} \, dx = -\frac{1}{a} \operatorname{tg}^{-1} \sinh ax + \frac{\sinh ax}{a}$
601.  $\int \frac{\cosh^2 ax}{\sinh ax} \, dx = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\cosh ax}{a}$
602.  $\int \frac{dx}{(1+\sinh ax) \cosh ax} = \frac{1}{2a} \ln \left( \frac{1+\sinh ax}{\cosh ax} \right) + \frac{1}{a} \operatorname{tg}^{-1} e^{ax}$
603.  $\int \frac{dx}{(\cosh ax \pm 1) \sinh ax} = \pm \frac{1}{2a(\cosh ax \pm 1)} \pm \frac{1}{2a} \ln \operatorname{tgh} \frac{ax}{2}$

INTEGRALES CON  $\operatorname{tgh} ax$

604.  $\int \operatorname{tgh} ax \, dx = \frac{1}{a} \ln \cosh ax$
605.  $\int \operatorname{tgh}^2 ax \, dx = -\frac{\operatorname{tgh} ax}{a} + x$
606.  $\int \operatorname{tgh}^3 ax \, dx = -\frac{\operatorname{tgh}^2 ax}{2a} + \frac{1}{a} \ln \cosh ax$
607.  $\int \operatorname{tgh}^n ax \operatorname{sech}^2 ax \, dx = \frac{\operatorname{tgh}^{n+1} ax}{(n+1)a}$
608.  $\int \frac{\sec h^2 ax}{\operatorname{tgh} ax} \, dx = \frac{1}{a} \ln \operatorname{tgh} ax$
609.  $\int \frac{dx}{\operatorname{tgh} ax} = \frac{1}{a} \ln \sinh ax$
610.  $\int x \operatorname{tgh} ax \, dx = \frac{1}{a^2} \left( \frac{(ax)^3}{3} - \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} - \dots + \frac{(-1)^{n-1} 2^{2n} (2^{2n}-1) B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right)$

$B_n$  es un  $n^\circ$  de Bernoulli tanto en 610 como en 611.

611.  $\int \frac{\operatorname{tgh} ax}{x} \, dx = ax - \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} - \dots + \frac{(2^{2n}-1) B_n (ax)^{2n-1}}{(2n-1)(2n)!} + \dots$
612.  $\int x \operatorname{tgh}^2 ax \, dx = -\frac{x \operatorname{tgh} ax}{a} + \frac{1}{a^2} \ln \cosh ax + \frac{x^2}{2}$
613.  $\int \frac{dx}{p+q \operatorname{tgh} ax} = \frac{px}{p^2-q^2} - \frac{q}{a(p^2-q^2)} \ln(q \sinh ax + p \cosh ax)$
614.  $\int \operatorname{tgh}^n ax \, dx = -\frac{\operatorname{tgh}^{n-1} ax}{(n-1)a} + \int \operatorname{tgh}^{n-2} ax \, dx$

INTEGRALES CON  $\operatorname{cotgh} ax$

615.  $\int \operatorname{cotgh} ax \, dx = \frac{1}{a} \ln \sinh ax$
- 30 616.  $\int \operatorname{cotgh}^2 ax \, dx = -\frac{\operatorname{cotgh} ax}{a} + x$

617.  $\int \cotgh^3 ax \, dx = -\frac{\cotgh^2 ax}{2a} + \frac{1}{a} \ln \sinh ax$
618.  $\int \cotgh^n ax \operatorname{cosech}^2 ax \, dx = -\frac{\cotgh^{n+1} ax}{(n+1)a}$
619.  $\int \frac{\operatorname{cosech}^2 ax}{\cotgh ax} dx = -\frac{1}{a} \ln \cotgh ax$
620.  $\int \frac{dx}{\cotgh ax} = \frac{1}{a} \ln \cosh ax$
621.  $\int x \cotgh ax \, dx = \frac{1}{a^2} \left( ax + \frac{(ax)^3}{9} - \frac{(ax)^5}{225} + \dots + \frac{(-1)^{n-1} 2^{2n} B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right)$   
*B<sub>n</sub> es n° de Bernoulli, tanto en 621 como en 622*
622.  $\int \frac{\cotgh ax}{x} dx = -\frac{1}{ax} + \frac{ax}{3} - \frac{(ax)^3}{135} + \dots + \frac{(-1)^n 2^{2n} B_n (ax)^{2n-1}}{(2n-1)!} + \dots$
623.  $\int x \cotgh^2 ax \, dx = -\frac{x \cotgh ax}{a} + \frac{1}{a^2} \ln \sinh ax + \frac{x^2}{2}$
624.  $\int \frac{dx}{p+q \cotgh ax} = \frac{px}{p^2-q^2} - \frac{q}{a(p^2-q^2)} \ln(p \sinh ax + q \cosh ax)$
625.  $\int \cotgh^n ax \, dx = -\frac{\cotgh^{n-1} ax}{(n-1)a} + \int \cotgh^{n-1} ax \, dx$

**INTEGRALES CON  $\operatorname{sech} ax$**

626.  $\int \operatorname{sech} ax \, dx = \frac{2}{a} \operatorname{tg}^{-1} e^{ax}$
627.  $\int \operatorname{sech}^2 ax \, dx = \frac{\operatorname{tgh} ax}{a}$
628.  $\int \operatorname{sech}^3 ax \, dx = \frac{\operatorname{sech} ax \operatorname{tgh} ax}{2a} + \frac{1}{2a} \operatorname{tg}^{-1} \sinh ax$
629.  $\int \operatorname{sech}^n ax \operatorname{tgh} ax \, dx = -\frac{\operatorname{sech}^{n-1} ax}{na}$
630.  $\int \frac{dx}{\operatorname{sech} ax} = \frac{\sinh ax}{a}$
631.  $\int x \operatorname{sech} ax \, dx = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} - \dots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\}$  *E<sub>n</sub> es n° de Euler*
632.  $\int \frac{\operatorname{sech} ax}{x} dx = \ln x - \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} - \frac{61(ax)^6}{4320} + \dots + \frac{(-1)^n E_n (ax)^{2n}}{2n(2n)!} + \dots$
633.  $\int x \operatorname{sech}^2 ax \, dx = \frac{x}{a} \operatorname{tgh} ax - \frac{1}{a^2} \ln \cosh ax$
634.  $\int \frac{dx}{q+p \operatorname{sech} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p+q \cosh ax}$  *Véase 582*
635.  $\int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \operatorname{tgh} ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx$

**INTEGRALES CON  $\operatorname{cosech} ax$**

636.  $\int \operatorname{cosech} ax \, dx = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2}$
637.  $\int \operatorname{cosech}^2 ax \, dx = -\frac{\cotgh ax}{a}$
638.  $\int \operatorname{cosech}^3 ax \, dx = -\frac{\operatorname{cosech} ax \cotgh ax}{2a} - \frac{1}{2a} \ln \operatorname{tgh} \frac{ax}{2}$
639.  $\int \operatorname{cosech}^n ax \cotgh ax \, dx = -\frac{\operatorname{cosech}^{n-1} ax}{na}$
640.  $\int \frac{dx}{\operatorname{cosech} ax} = \frac{\cosh ax}{a}$
641.  $\int x \operatorname{cosech} ax \, dx = \frac{1}{a^2} \left\{ ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} - \dots + \frac{(-1)^n 2(2^{2n-1}-1) B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}$

*B<sub>n</sub> es n° de Bernoulli*

$$642. \int \frac{\operatorname{cosech} ax}{x} dx = -\frac{1}{ax} - \frac{ax}{6} + \frac{7(ax)^3}{1080} - \dots + \frac{(-1)^n 2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \dots$$

$$643. \int x \operatorname{cosech}^2 ax dx = -\frac{x}{a} \operatorname{cotgh} ax + \frac{1}{a^2} \ln \sinh ax$$

$$644. \int \frac{dx}{q+p \operatorname{cosech} ax} = \frac{x}{p} - \frac{p}{q} \int \frac{dx}{p+q \sinh ax} \quad \text{Véase 554}$$

$$645. \int \operatorname{cosech}^n ax dx = -\frac{\operatorname{cosech}^{n-2} ax \operatorname{cotgh} ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosech}^{n-2} ax dx$$

**INTEGRALES DE FUNCIONES HIPERBOLICAS INVERSAS**

$$646. \int \sinh^{-1} \frac{x}{a} dx = x \sinh^{-1} \frac{x}{a} - \sqrt{a^2 + x^2}$$

$$647. \int x \sinh^{-1} \frac{x}{a} dx = \left(\frac{x^2}{2} + \frac{a^2}{4}\right) \sinh^{-1} \frac{x}{a} - \frac{x\sqrt{a^2+x^2}}{4}$$

$$648. \int x^2 \sinh^{-1} \frac{x}{a} dx = \frac{x^3}{3} \sinh^{-1} \frac{x}{a} + \frac{(-x^2+2a^2)\sqrt{a^2+x^2}}{9}$$

$$649. \int x^m \sinh^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \sinh^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{a^2+x^2}} dx$$

$$650. \int \frac{\sinh^{-1} x/a}{x} dx = \begin{cases} \frac{x}{a} - \frac{(x/a)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 (x/a)^5}{2 \cdot 4 \cdot 5 \cdot 5} - \frac{1 \cdot 3 \cdot 5 (x/a)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots & |x| < a \\ \frac{\ln^2(2x/a)}{2} - \frac{(a/x)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (a/x)^4}{2 \cdot 4 \cdot 4 \cdot 4} - \frac{1 \cdot 3 \cdot 5 (a/x)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} + \dots & x > a \\ \frac{-\ln^2(-2x/a)}{2} + \frac{(a/x)^2}{2 \cdot 2 \cdot 2} - \frac{1 \cdot 3 (a/x)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \frac{1 \cdot 3 \cdot 5 (a/x)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} - \dots & x < -a \end{cases}$$

$$651. \int \frac{\sinh^{-1} x/a}{x^2} dx = -\frac{\sinh^{-1} x/a}{x} - \frac{1}{a} \ln \left( \frac{a+\sqrt{a^2+x^2}}{x} \right)$$

$$652. \int \cosh^{-1} \frac{x}{a} dx = \begin{cases} x \cosh^{-1} \frac{x}{a} - \sqrt{x^2 - a^2}; & \cosh^{-1} \frac{x}{a} > 0 \\ x \cosh^{-1} \frac{x}{a} + \sqrt{x^2 - a^2}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$653. \int x \cosh^{-1} \frac{x}{a} dx = \begin{cases} \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \cosh^{-1} \frac{x}{a} - \frac{x\sqrt{x^2-a^2}}{4}; & \cosh^{-1} \frac{x}{a} > 0 \\ \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \cosh^{-1} \frac{x}{a} + \frac{x\sqrt{x^2-a^2}}{4}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$654. \int x^2 \cosh^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^3}{3} \cosh^{-1} \frac{x}{a} - \frac{x+a\sqrt{x^2-a^2}}{9}; & \cosh^{-1} \frac{x}{a} > 0 \\ \frac{x^3}{3} \cosh^{-1} \frac{x}{a} + \frac{(x^2+2a^2)\sqrt{x^2-a^2}}{9}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$655. \int x^m \cosh^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \cosh^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2-a^2}} dx; & \cosh^{-1} \frac{x}{a} > 0 \\ \frac{x^{m+1}}{m+1} \cosh^{-1} \frac{x}{a} + \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2-a^2}} dx; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

$$656. \int \frac{\cosh^{-1} x/a}{x} dx = \pm \left( \frac{\ln^2(2a)}{2} + \frac{(a/x)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (a/x)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \frac{1 \cdot 3 \cdot 5 (a/x)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} + \dots \right) \\ + \text{si } \cosh^{-1} \frac{x}{a} > 0; - \text{si } \cosh^{-1} \frac{x}{a} < 0$$

$$657. \int \frac{\cosh^{-1}(x/a)}{x^2} dx = -\frac{\cosh^{-1}(x/a)}{x} \mp \frac{1}{a} \ln \left( \frac{a+\sqrt{a^2+x^2}}{x} \right) \\ - \text{si } \cosh^{-1} \frac{x}{a} > 0; + \text{si } \cosh^{-1} \frac{x}{a} < 0$$

$$658. \int \operatorname{tgh}^{-1} \frac{x}{a} dx = x \operatorname{tgh}^{-1} \frac{x}{a} + \frac{a}{2} \ln(a^2 - x^2)$$

$$659. \int x \operatorname{tgh}^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 - a^2) \operatorname{tgh}^{-1} \frac{x}{a} + \frac{ax}{2}$$

$$660. \int x^2 \operatorname{tgh}^{-1} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{tgh}^{-1} \frac{x}{a} + \frac{ax^2}{6} + \frac{a^3}{6} \ln(a^2 - x^2)$$

$$661. \int x^m \operatorname{tgh}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{tgh}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{a^2-x^2} dx$$

$$32 \quad 662. \int \frac{\operatorname{tgh}^{-1}(x/a)}{x} dx = \frac{x}{a} + \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} + \frac{(x/a)^7}{7^2} + \dots$$

663.  $\int \frac{\operatorname{tgh}^{-1}(x/a)}{x^2} dx = -\frac{1}{x} \operatorname{tgh}^{-1} \frac{x}{a} + \frac{1}{2a} \ln \left( \frac{x^2}{a^2 - x^2} \right)$
664.  $\int \cot \operatorname{gh}^{-1} \frac{x}{a} dx = x \cot \operatorname{gh}^{-1} \frac{x}{a} + \frac{a}{2} \ln(x^2 - a^2)$
665.  $\int x \cot \operatorname{gh}^{-1} \frac{x}{a} dx = \frac{1}{2}(x^2 - a^2) \cot \operatorname{gh}^{-1} \frac{x}{a} + \frac{ax}{2}$
666.  $\int x^2 \cot \operatorname{gh}^{-1} \frac{x}{a} dx = \frac{ax^2}{6} + \frac{x^3}{3} \cot \operatorname{gh}^{-1} \frac{x}{a} + \frac{a^3}{6} \ln(x^2 - a^2)$
667.  $\int x^m \cot \operatorname{gh}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \cot \operatorname{gh}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{a^2 - x^2} dx$
668.  $\int \frac{\cot \operatorname{gh}^{-1}(x/a)}{x} dx = -\left( \frac{a}{x} + \frac{(a/x)^3}{3^2} + \frac{(a/x)^5}{5^2} + \frac{(a/x)^7}{7^2} + \dots \right)$
669.  $\int \frac{\cot \operatorname{gh}^{-1}(x/a)}{x^2} dx = -\frac{1}{x} \cot \operatorname{gh}^{-1} \frac{x}{a} + \frac{1}{2a} \ln \left( \frac{x^2}{x^2 - a^2} \right)$
670.  $\int \operatorname{sech}^{-1} \frac{x}{a} dx = \begin{cases} x \operatorname{sech}^{-1} \frac{x}{a} + a \operatorname{sen}^{-1}(x/a); & \operatorname{sech}^{-1} \frac{x}{a} > 0 \\ x \operatorname{sech}^{-1} \frac{x}{a} - a \operatorname{sen}^{-1}(x/a); & \operatorname{sech}^{-1} \frac{x}{a} < 0 \end{cases}$
671.  $\int x \operatorname{sech}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x}{2} \operatorname{sech}^{-1} \frac{x}{a} - \frac{a\sqrt{a^2 - x^2}}{2}; & \operatorname{sech}^{-1} \frac{x}{a} > 0 \\ \frac{x}{2} \operatorname{sech}^{-1} \frac{x}{a} + \frac{a\sqrt{a^2 - x^2}}{2}; & \operatorname{sech}^{-1} \frac{x}{a} < 0 \end{cases}$
672.  $\int x^m \operatorname{sech}^{-1} \frac{x}{a} dx = \begin{cases} \frac{x^{m+1}}{m+1} \operatorname{sech}^{-1} \frac{x}{a} + \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{a^2 - x^2}}; & \operatorname{sech}^{-1} \frac{x}{a} > 0 \\ \frac{x^{m+1}}{m+1} \operatorname{sech}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{a^2 - x^2}}; & \operatorname{sech}^{-1} \frac{x}{a} < 0 \end{cases}$
673.  $\int \frac{\operatorname{sech}^{-1}(x/a)}{x} dx = \mp \left( \frac{\ln(x/a) \ln(4x/a)}{2} + \frac{(x/a)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (x/a)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \frac{1 \cdot 3 \cdot 5 (x/a)^6}{2 \cdot 4 \cdot 6 \cdot 6 \cdot 6} + \dots \right)$
674.  $\int \operatorname{cosech}^{-1} \frac{x}{a} dx = x \operatorname{cosech}^{-1} \frac{x}{a} \pm a \operatorname{senh}^{-1} \frac{x}{a}; \quad [+ \text{ si } x > 0, - \text{ si } x < 0]$
675.  $\int x \operatorname{cosech}^{-1} \frac{x}{a} dx = \frac{x^2}{2} \operatorname{cosech}^{-1} \frac{x}{a} \pm \frac{a\sqrt{x^2 + a^2}}{2}; \quad [+ \text{ si } x > 0, - \text{ si } x < 0]$
676.  $\int x^m \operatorname{cosech}^{-1} \frac{x}{a} dx = \frac{x^{m+1}}{m+1} \operatorname{cosech}^{-1} \frac{x}{a} \pm \frac{a}{m+1} \int \frac{x^m dx}{\sqrt{x^2 + a^2}}; \quad [+ \text{ si } x > 0, - \text{ si } x < 0]$
677.  $\int \frac{\operatorname{cosech}^{-1}(x/a)}{x} dx = \begin{cases} \frac{\ln(x/a) \ln(x/a)}{2} + \frac{(x/a)^2}{2 \cdot 2 \cdot 2} - \frac{1 \cdot 3 (x/a)^4}{2 \cdot 4 \cdot 4 \cdot 4} + \dots; & 0 < x < a \\ \frac{\ln(-x/a) \ln(-x/a)}{2} - \frac{(x/a)^2}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 3 (x/a)^4}{2 \cdot 4 \cdot 4 \cdot 4} - \dots; & -a < x < 0 \\ -\frac{a}{x} + \frac{(a/x)^3}{2 \cdot 3 \cdot 3} - \frac{1 \cdot 3 (a/x)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \dots; & |x| > a \end{cases}$

## INTEGRALES DEFINIDAS

### PROPIEDADES

678.  $\int_a^b [k f(x) + r g(x) - n h(x)] dx = k \int_a^b f(x) dx + r \int_a^b g(x) dx - n \int_a^b h(x) dx;$   
 $a, b, k, r, n \in \mathbb{R}$
679.  $\int_a^a f(x) dx = 0$
680.  $\int_a^b f(x) dx = -\int_b^a f(x) dx$
681.  $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$
682.  $\int_a^b f(x) dx = f(c)(b-a)$  Para algún  $c$  tal que  $a < c < b$ ,  $f$  continua en  $[a, b]$

683. Si  $f'(x) = F(x) = \int_a^b f(x) dx = F(b) - F(a)$  (Regla de Barrow)

### INTEGRALES IMPROPIAS

684.  $\int_a^{+\infty} f(x) dx = \lim_{z \rightarrow +\infty} \int_a^z f(x) dx$

685.  $\int_{-\infty}^b f(x) dx = \lim_{\substack{b \rightarrow +\infty \\ a \rightarrow -\infty}} \int_a^b f(x) dx$

686.  $\int_a^b f(x) dx = \lim_{\theta \rightarrow 0} \int_a^{b-\theta} f(x) dx$  Si  $b$  es punto singular de  $f, \theta > 0$

687.  $\int_a^b f(x) dx = \lim_{\theta \rightarrow 0} \int_{a+\theta}^b f(x) dx$  Si  $a$  es punto singular de  $f, \theta > 0$

### INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES TRIGONOMETRICAS

688.  $\int_0^\pi \sin nx \sin kx dx = \int_0^\pi \cos nx \cos kx dx = \begin{cases} 0 & \text{si } n \neq k \\ \frac{\pi}{2} & \text{si } n = k \end{cases} ; n, k \in \mathbb{Z}$

689.  $\int_0^\pi \sin kx \cos nx dx = \begin{cases} 0 & \text{si } n+k \text{ es impar} \\ \frac{2k}{k^2-n^2} & \text{si } n+k \text{ es par} \end{cases} ; n, k \in \mathbb{Z}$

690.  $\int_0^{\pi/2} \sin^2 x dx = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$

691.  $\int_0^{\pi/2} \sin^{2k} x dx = \int_0^{\pi/2} \cos^{2k} x dx = \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots 2k} - \frac{\pi}{2} \quad k \in \mathbb{N}$

692.  $\int_0^{\pi/2} \sin^{2k+1} x dx = \int_0^{\pi/2} \cos^{2k+1} x dx = \frac{2 \cdot 4 \cdot 6 \cdots 2k}{1 \cdot 3 \cdot 5 \cdots (2k+1)} \quad k \in \mathbb{N}$

693.  $\int \sin^{2p-1} x \cos^{2q-1} x dx = \frac{\Gamma(p)\Gamma(q)}{2 \cdot \Gamma(p+q)} \quad \Gamma: \text{Función Gamma (ver apéndice)}$

694.  $\int_0^\infty \frac{\sin kx}{x} dx = \begin{cases} \frac{\pi}{2} & k > 0 \\ 0 & k = 0 \\ -\frac{\pi}{2} & k < 0 \end{cases}$

695.  $\int_0^\infty \frac{\sin px \cos qx}{x} dx = \begin{cases} \pi/2 & 0 < p < q \\ 0 & 0 < q < p \\ \pi/4 & p = q > 0 \end{cases}$

696.  $\int_0^\infty \frac{\sin px \cos qx}{x^2} dx = \begin{cases} p\pi/2 & 0 < p < q \\ q\pi/2 & 0 < q \leq p \end{cases}$

697.  $\int_0^\infty \frac{\sin^2 px}{x^2} dx = \frac{p\pi}{2}$

698.  $\int_0^\infty \frac{1 - \cos px}{x^2} dx = \frac{p\pi}{2}$

699.  $\int_0^\infty \frac{\cos px - \cos qx}{x} dx = \ln \frac{q}{p}$

700.  $\int_0^\infty \frac{\cos px - \cos qx}{x^2} dx = \frac{(q-p)\pi}{2}$

701.  $\int_0^\infty \frac{\cos px}{x^2+a^2} dx = \frac{\pi}{2a} e^{-na}$

34 702.  $\int_0^\infty \frac{x \sin px}{x^2+a^2} dx = \frac{\pi}{2} e^{-na}$

703.  $\int_0^{\infty} \frac{\operatorname{sen} nx}{x(x^2+a^2)} dx = \frac{\pi}{2a^2} (1 - e^{-na})$
704.  $\int_0^{2\pi} \frac{dx}{a+b \operatorname{sen} x} = \frac{2\pi}{\sqrt{a^2-b^2}}$
705.  $\int_0^{\pi/2} \frac{dx}{a+b \cos x} = \frac{\cos^{-1}(b/a)}{\sqrt{a^2-b^2}}$
706.  $\int_0^{2\pi} \frac{dx}{(a+b \operatorname{sen} x)^2} = \int_0^{2\pi} \frac{dx}{(a+b \cos x)^2} = \frac{2\pi a}{\sqrt{(a^2-b^2)^3}}$
707.  $\int_0^{2\pi} \frac{dx}{1-2a \cos x+a^2} = \frac{2\pi}{1-a^2} \quad 0 < a < 1$
708.  $\int_0^{\pi} \frac{x \operatorname{sen} x dx}{1-2a \cos x+a^2} = \begin{cases} \frac{\pi}{a} \ln(1+a) & \text{si } |a| < 1 \\ \pi \ln(1+\frac{1}{a}) & |a| > 1 \end{cases}$
709.  $\int_0^{\pi} \frac{\cos kx dx}{1-2a \cos x+a^2} = \frac{\pi a^k}{1-a^2} \quad \text{si } |a| < 1, k \in \mathbb{N}$
710.  $\int_0^{\infty} \operatorname{sen} ax^2 dx = \int_0^{\infty} \cos ax^2 dx = \sqrt{\frac{\pi}{8a}}$
711.  $\int_0^{\infty} \operatorname{sen} ax^n dx = \frac{\Gamma(1/n)}{n a^{1/n}} \operatorname{sen} \frac{\pi}{2n}; \quad n > 1, \Gamma: \text{Función Gamma (Ver apéndice)}$
712.  $\int_0^{\infty} \cos ax^n dx = \frac{\Gamma(1/n)}{n a^{1/n}} \cos \frac{\pi}{2n}; \quad n > 1$
713.  $\int_0^{\infty} \frac{\operatorname{sen} x}{\sqrt{x}} dx = \int_0^{\infty} \frac{\cos x}{\sqrt{x}} dx = \sqrt{\frac{\pi}{2}}$
714.  $\int_0^{\infty} \frac{\operatorname{sen} x}{x^p} dx = \frac{\pi}{2\Gamma(p) \operatorname{sen}(p\pi/2)}; \quad 0 < p < 1$
715.  $\int_0^{\infty} \frac{\cos x}{x^p} dx = \frac{\pi}{2\Gamma(p) \operatorname{sen}(p\pi/2)}; \quad 0 < p < 1$
716.  $\int_0^{\infty} \operatorname{sen} ax^2 \cos 2bx dx = \sqrt{\frac{\pi}{8a}} (\cos \frac{b^2}{a} + \operatorname{sen} \frac{b^2}{a})$
717.  $\int_0^{\infty} \cos ax^2 \operatorname{sen} 2bx dx = \sqrt{\frac{\pi}{8a}} (\cos \frac{b^2}{a} + \operatorname{sen} \frac{b^2}{a})$
718.  $\int_0^{\infty} \frac{\operatorname{sen}^3 x}{x^3} dx = \frac{3\pi}{8}$
719.  $\int_0^{\infty} \frac{\operatorname{sen}^4 x}{x^4} dx = \frac{\pi}{3}$
720.  $\int_0^{\infty} \frac{\operatorname{tg} x}{x} dx = \frac{\pi}{2}$
721.  $\int_0^{\pi/2} \frac{dx}{1+\operatorname{tg}^n x} = \frac{\pi}{4}$
722.  $\int_0^{\pi/2} \frac{x dx}{\operatorname{sen} x} = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2}$
723.  $\int_0^1 \frac{\operatorname{tg}^{-1} x}{x} dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2}$
724.  $\int_0^1 \frac{\operatorname{sen}^{-1} x}{x} dx = \frac{\pi}{2} \ln 2$
725.  $\int_0^1 \frac{1-\cos x}{x} dx - \int_0^{\infty} \frac{\cos x}{x} dx = \gamma, \quad \gamma: \text{Constante de Euler}$
726.  $\int_0^{\infty} \left( \frac{1}{1+x^2} - \cos x \right) \frac{dx}{x} = \gamma$
727.  $\int_0^{\infty} \frac{\operatorname{tg}^{-1} px - \operatorname{tg}^{-1} qx}{x} dx = \frac{\pi}{2} \ln \frac{p}{q}$

**INTEGRALES DEFINIDAS O IMPROPIAS  
DE FUNCIONES RACIONALES E IRRACIONALES**

$$728. \int_b^{\infty} \frac{dx}{x^2+a^2} = \frac{\pi}{2a}$$

$$729. \int_b^{\infty} \frac{x^{p-1} dx}{1+x} = \frac{\pi}{\operatorname{sen} p\pi}; \quad 0 < p < 1$$

$$730. \int_b^{\infty} \frac{x^m dx}{x^n+a^n} = \frac{\pi a^{m-n+1}}{n \operatorname{sen} \left(\frac{m+1}{n}\pi\right)}; \quad 0 < m+1 < n$$

$$731. \int_b^{\infty} \frac{x^m dx}{1+2x \cos \beta + x^2} = \frac{\pi}{\operatorname{sen} m\alpha} \cdot \frac{\operatorname{sen} m\beta}{\operatorname{sen} \beta}$$

$$732. \int_b^{\infty} \frac{dx}{\sqrt{a^2-x^2}} = \frac{\pi}{2}$$

$$733. \int_b^{\infty} \sqrt{a^2-x^2} dx = \frac{\pi a^2}{4}$$

$$734. \int_b^{\infty} x^m (a^n - x^n)^p dx = \frac{a^{m+np+1}}{n} \cdot \frac{\Gamma(m+1/n) \cdot \Gamma(p+1)}{\Gamma(m+1/n+p+1)} \quad \Gamma: \text{Función Gamma}$$

$$735. \int_b^{\infty} \frac{x^m dx}{(a^n+x^n)^r} = \frac{(-1)^{r-1} \pi a^{m-nr+1}}{n \operatorname{sen} \left(\frac{m+1}{n}\pi\right) \cdot (r-1)!} \cdot \frac{\Gamma(m+1/n)}{\Gamma(m+1/n-r+1)}; \quad 0 < m+1 < nr$$

**INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES  
EXPONENCIALES**

$$736. \int_b^{\infty} e^{-ax} \cos bx dx = \frac{a}{a^2+b^2}$$

$$737. \int_b^{\infty} e^{-ax} \operatorname{sen} bx dx = \frac{b}{a^2+b^2}$$

$$738. \int_0^{\infty} \frac{e^{-ax} \operatorname{sen} bx}{x} dx = \operatorname{tg}^{-1} \frac{b}{a}$$

$$739. \int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} dx = \ln \frac{b}{a}$$

$$740. \int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$741. \int_0^{\infty} e^{-ax^2} \cos bx dx = \frac{1}{2} \sqrt{\frac{\pi}{2}} e^{-(b^2/4a)}$$

$$742. \int_0^{\infty} e^{-(ax^2+bx+c)} dx = \frac{1}{2} \sqrt{\frac{\pi}{2}} e^{(b^2-4a)/4a} \cdot f_{\operatorname{cer}} \frac{b}{2\sqrt{a}}; \quad \text{Siendo } f_{\operatorname{cer}}(p) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-k^2} dk$$

$$743. \int_0^{\infty} e^{-(ax^2+bx+c)} dx = \sqrt{\frac{\pi}{a}} e^{(b^2-4a)/4a}$$

$$744. \int_0^{\infty} x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}}; \quad \Gamma: \text{Función Gamma}$$

$$745. \int_0^{\infty} x^m e^{-ax^2} dx = \frac{\Gamma(m+1/2)}{2 a^{(m+1/2)}}$$

$$746. \int_0^{\infty} e^{-(ax^2+b/m^2)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-2\sqrt{ab}}$$

$$747. \int_0^{\infty} \frac{x dx}{e^x-1} = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$748. \int_0^{\infty} \frac{x^{n-1} dx}{e^x-1} = \Gamma(n+1) \sum_{k=1}^{\infty} \frac{1}{k^n}; \quad \Gamma: \text{Función Gamma}$$

Si  $n$  es par esta serie se puede hallar con ayuda de los números de Bernoulli (ver apéndice)

$$749. \int_0^{\infty} \frac{x dx}{e^x + 1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

$$750. \int_0^{\infty} \frac{x^{n-1} dx}{e^x + 1} = \Gamma(n+1) \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^n} \quad \Gamma : \text{Función Gamma}$$

$$751. \int_0^{\infty} \frac{\operatorname{sen} mx}{e^{2\pi x} - 1} dx = \frac{1}{4} \cot \frac{m}{2} - \frac{1}{2m}$$

$$752. \int_0^{\infty} \left( \frac{1}{1+x} - e^{-x} \right) \frac{dx}{x} = \gamma \quad \gamma : \text{Constante de Euler}$$

$$753. \int_0^{\infty} \frac{e^{-x^2} - e^{-x}}{x} dx = \frac{\gamma}{2}$$

$$754. \int_0^{\infty} \left( \frac{1}{e^{-x}-1} - \frac{e^{-x}}{x} \right) dx = \gamma \quad \gamma : \text{Constante de Euler}$$

$$755. \int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x \operatorname{sech} px} dx = \frac{1}{2} \ln \frac{b^2 + p^2}{a^2 + p^2}$$

$$756. \int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x \operatorname{cosec} px} dx = \operatorname{tg}^{-1} \frac{b}{p} - \operatorname{tg}^{-1} \frac{a}{p}$$

$$757. \int_0^{\infty} \frac{e^{-ax}(1-\cos x)}{x^2} dx = \cot g^{-1} a - \frac{a}{2} \ln(a^2 + 1)$$

#### INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES LOGARITMICAS

$$758. \int_0^1 x^m \ln^n x dx = \frac{(-1)^n n!}{(m+1)^{n+1}}; \quad m > -1, n \in N_0$$

$$759. \int_0^1 x^m \ln^n x dx = \frac{(-1)^n \Gamma(n+1)}{(m+1)^{n+1}}; \quad m > -1, n \notin N_0, \Gamma : \text{Función Gamma}$$

$$760. \int_0^1 \frac{\ln x}{1+x} dx = -\frac{\pi^2}{12}$$

$$761. \int_0^1 \frac{\ln x}{1-x} dx = -\frac{\pi^2}{6}$$

$$762. \int_0^1 \frac{\ln(1+x)}{x} dx = \frac{\pi^2}{12}$$

$$763. \int_0^1 \frac{\ln(1-x)}{x} dx = -\frac{\pi^2}{6}$$

$$764. \int_0^1 \ln x \ln(1+x) dx = 2(1 - \ln 2) - \frac{\pi^2}{12}$$

$$765. \int_0^1 \ln x \ln(1-x) dx = 2 - \frac{\pi^2}{6}$$

$$766. \int_0^{\infty} \frac{x^{p-1} \ln x}{1+x} dx = -\pi^2 \operatorname{cosec} p\pi \cdot \cot g p\pi; \quad 0 < p < 1$$

$$767. \int_0^1 \frac{x^m - x^n}{\ln x} dx = \ln \frac{m+1}{n+1}$$

$$768. \int_0^{\infty} e^{-x} \ln x dx = -\gamma \quad \gamma : \text{Constante de Euler}$$

$$769. \int_0^{\infty} e^{-x^2} \ln x dx = -\frac{\sqrt{\pi}}{4} (\gamma + 2 \ln 2)$$

$$770. \int_0^{\infty} \ln \frac{e^x + 1}{e^x - 1} dx = \frac{\pi^2}{4}$$

$$771. \int_0^{\pi/2} \ln \operatorname{sen} x dx = \int_0^{\pi/2} \ln \cos x dx = -\frac{\pi \ln 2}{2}$$

$$772. \int_0^{\pi/2} (\ln \operatorname{sen} x)^2 dx = \int_0^{\pi/2} (\ln \cos x)^2 dx = \frac{\pi^2}{4} + \frac{\pi \ln^2 2}{2}$$

$$773. \int_0^{\pi} x \ln \operatorname{sen} x dx = -\frac{\pi^2 \ln 2}{2}$$

$$774. \int_0^{\pi/2} \operatorname{sen} x \ln \operatorname{sen} x dx = \ln \frac{2}{e}$$

$$775. \int_0^{2\pi} \ln(a + b \operatorname{sen} x) dx = \int_0^{2\pi} \ln(a + b \cos x) dx = 2\pi \ln(a + \sqrt{a^2 - b^2})$$

$$776. \int_0^{\pi} \ln(a + b \cos x) dx = \pi \ln \frac{a + \sqrt{a^2 - b^2}}{2}$$

$$777. \int_0^{\pi} \ln(a^2 + 2ab \cos x + b^2) dx = \begin{cases} 2\pi \ln b; & \text{si } 0 < a \leq b \\ 2\pi \ln a; & \text{si } 0 < b \leq a \end{cases}$$

$$778. \int_0^{\pi/4} \ln(1 + \operatorname{tg} x) dx = \frac{\pi \ln 2}{8}$$

$$779. \int_0^{\pi/2} \sec x \ln \left( \frac{1+b \cos x}{1+a \cos x} \right) dx = \frac{(\cos^{-1} a)^2 - (\cos^{-1} b)^2}{2}$$

$$780. \int_0^a \ln \left( 2 \operatorname{sen} \frac{x}{2} \right) dx = -\sum_{n=1}^{\infty} \frac{\operatorname{sen} an}{n^2}$$

### INTEGRALES IMPROPIAS DE FUNCIONES HIPERBOLICAS

$$781. \int_0^{\infty} \frac{\operatorname{sen} ax}{\operatorname{senh} bx} dx = \frac{\pi}{2b} \operatorname{tgh} \frac{a\pi}{2b}$$

$$782. \int_0^{\infty} \frac{\cos ax}{\cosh} dx = \frac{\pi}{2b} \operatorname{sec} h \frac{a\pi}{2b}$$

$$783. \int_0^{\infty} \frac{x dx}{\operatorname{senh} ax} = \frac{\pi^2}{4a^2}$$

$$784. \int_0^{\infty} \frac{x^n dx}{\operatorname{senh} ax} = \frac{2^{n+1}-1}{2^{n+1} a^{n+1}} \Gamma(n+1) \sum_{k=1}^{\infty} \frac{1}{k^{n+1}}; \quad \Gamma: \text{Función Gamma}$$

$$785. \int_0^{\infty} \frac{\operatorname{senh} ax}{e^{bx} + 1} = \frac{\pi}{2b} \operatorname{cosec} \frac{a\pi}{b} - \frac{1}{2a}$$

$$786. \int_0^{\infty} \frac{\operatorname{senh} ax}{e^{bx} - 1} dx = \frac{1}{2a} - \frac{\pi}{2b} \operatorname{cot} g \frac{a\pi}{b}$$

### APENDICE

#### FUNCION GAMMA

Definición:  $\Gamma(n) = \int_0^{\infty} t^{n-1} e^{-t} dt; \quad n > 0$

Formula de recurrencia:  $\Gamma(n+1) = n\Gamma(n)$

Si  $n \in \mathbb{N} \Rightarrow \Gamma(n+1) = n!$  Si  $n < 0 \Rightarrow \Gamma(n) = \frac{\Gamma(n+1)}{n}$

Propiedades: a)  $\Gamma(p)\Gamma(1-p) = \frac{\pi}{\operatorname{sen} p\pi}$  b)  $\frac{2^{2p-1}}{\sqrt{\pi}} = \frac{\Gamma(2p)}{\Gamma(p)\Gamma(p+\frac{1}{2})}$

#### FUNCION BETA

Definición:  $B(m, n) = \int_0^1 t^{m-1} (1-t)^{n-1} dt; \quad m > 0, n > 0$

**38** Relación con la función Gamma:  $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Otras formas de expresar la función Beta:  $B(m, n) = B(n, m) =$

$$= 2 \int_0^{\pi/2} \sin^{2m-1} x \cos^{2n-1} x dx = \int_0^{\infty} u^{m-1} (1+u)^{-m-n} du = r^n (r+1)^m \int_0^1 \frac{t^{m-1} (1-t)^{n-1}}{(r+t)^{m+n}} dt$$

**NUMEROS DE BERNOULLI Y EULER**

Definición:  $B(m, n) = \int_0^1 t^{m-1} (1-t)^{n-1} dt; \quad m > 0; n > 0$

Relación con la función Gamma:  $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Otras formas de expresar la función Beta:  $B(m, n) = B(n, m) =$

$$= 2 \int_0^{\pi/2} \sin^{2m-1} x \cos^{2n-1} x dx = \int_0^{\infty} u^{m-1} (1+u)^{-m-n} du = r^n (r+1)^m \int_0^1 \frac{t^{m-1} (1-t)^{n-1}}{(r+t)^{m+n}} dt$$

**NUMEROS DE BERNOULLI Y EULER**

a) Bernoulli: los números  $B_1; B_2; B_3; \dots$  se definen por las series:

$$\frac{x}{e^x - 1} = 1 - \frac{x}{2} + \frac{B_1 x^2}{2!} + \frac{B_2 x^4}{4!} + \frac{B_3 x^6}{6!} + \dots \quad |x| < 2\pi$$

$$\text{ó también } 1 - \frac{x}{2} \operatorname{cotg}\left(\frac{x}{2}\right) = \frac{B_1 x^2}{2!} + \frac{B_2 x^4}{4!} + \frac{B_3 x^6}{6!} + \dots \quad |x| < \pi$$

b) Euler: los números de Euler  $E_1; E_2; E_3; \dots$  se definen por las series:

$$\operatorname{sech} x = 1 - \frac{E_1 x^2}{2!} + \frac{E_2 x^4}{4!} - \frac{E_3 x^6}{6!} + \dots \quad |x| < \pi/2$$

$$\sec x = 1 + \frac{E_1 x^2}{2!} + \frac{E_2 x^4}{4!} + \frac{E_3 x^6}{6!} + \dots \quad |x| < \pi/2$$

Tabla de algunos números  $B_n$  y  $E_n$

n	$B_n$	$E_n$
1	1/6	1
2	1/30	5
3	1/42	61
4	1/30	1385
5	5/66	50521
6	691/2730	2702765
7	7/6	199360981
8	3617/510	19391512145
9	43867/798	2404879675441
10	174611/330	370371188237525





