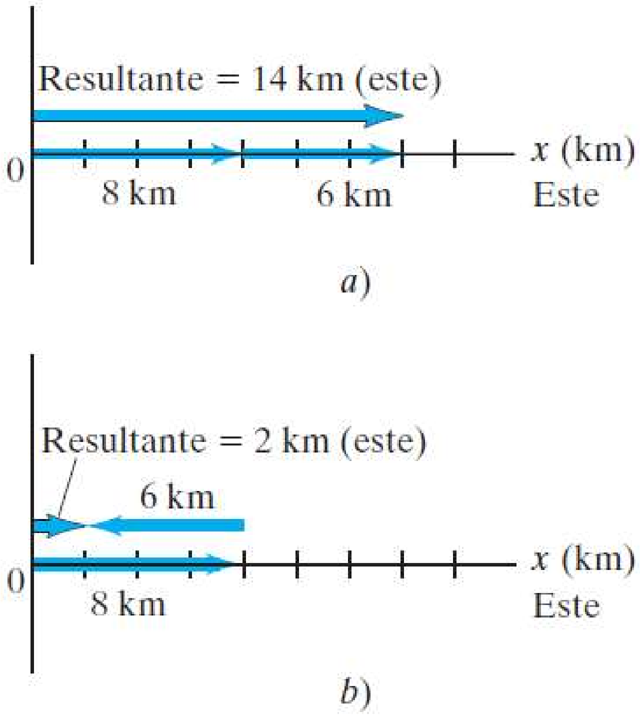
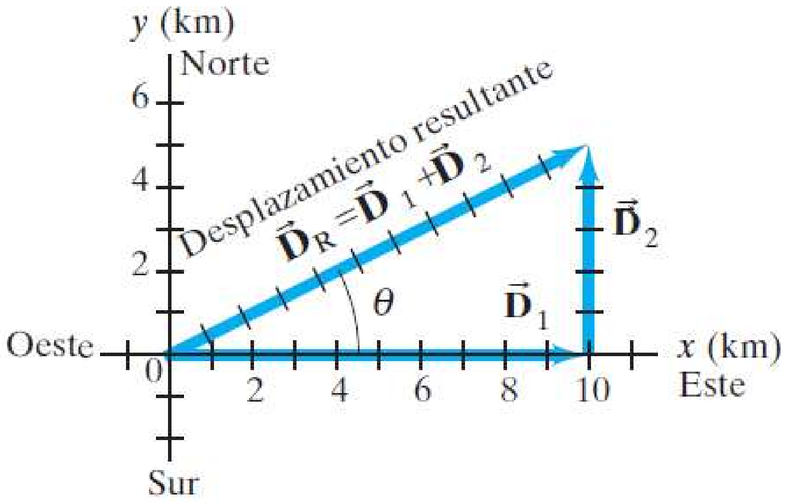
CINEMÁTICA EN 2D – REPASO VECTORES

Trabajar con vectores en forma gráfica:

Combinación de vectores en una dimensión.

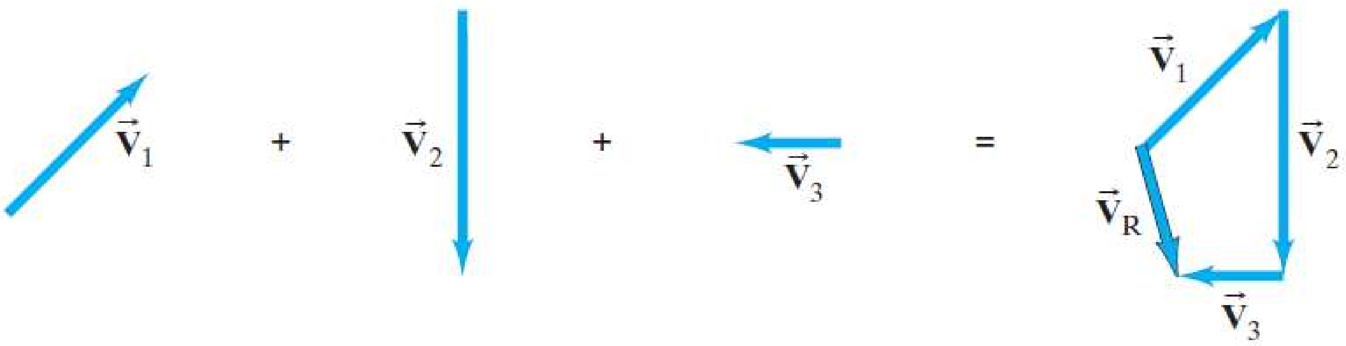


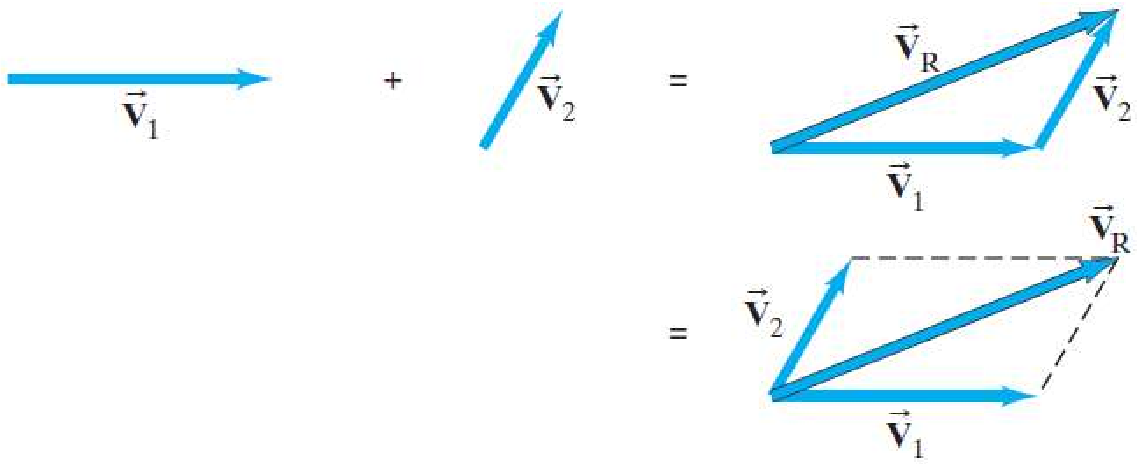


$$\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2$$

VER PROBLEMAS CUADERNILLO MÓDULO INGRESO

El vector resultante no se ve alterado por el orden en que se sumen los mismos





(a) Cola a punta

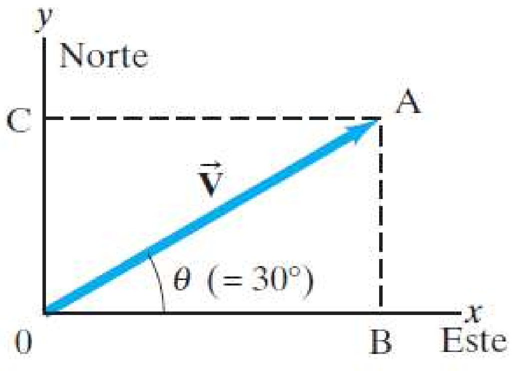
(b) Paralelogramo

Restar vectores:

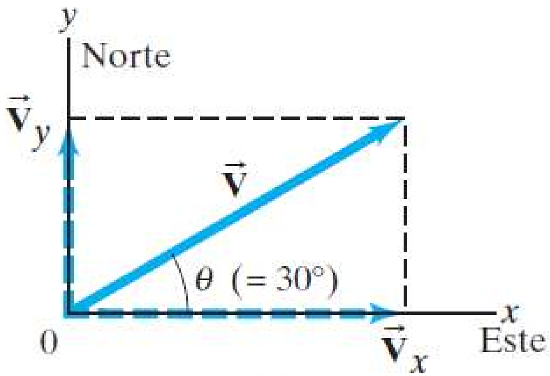
$$\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = \vec{\mathbf{V}}_2 + (-\vec{\mathbf{V}}_1)$$

Trabajar con vectores por componentes:

$$\boxed{\begin{aligned} V_y &= V \operatorname{sen}\theta \\ V_x &= V \cos\theta \end{aligned}}$$



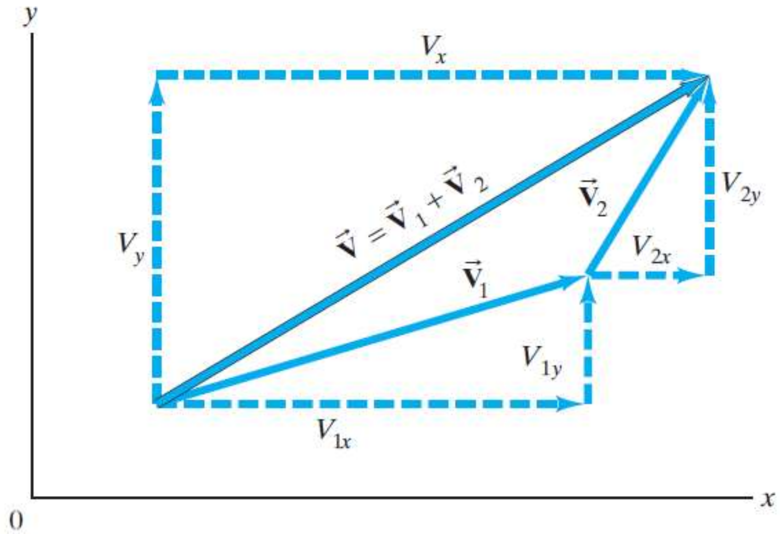
a)



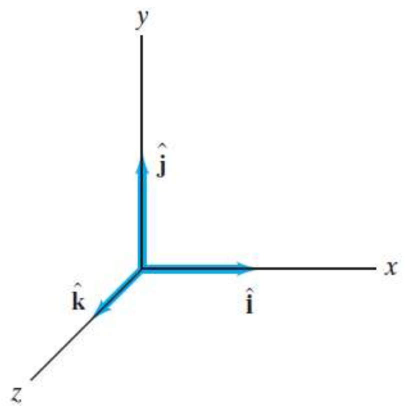
b)

$$\boxed{\begin{aligned} \operatorname{sen}\theta &= \frac{V_y}{V} \\ \cos\theta &= \frac{V_x}{V} \\ \tan\theta &= \frac{V_y}{V_x} \\ V^2 &= V_x^2 + V_y^2 \end{aligned}}$$

$$\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2, \text{ implica que}$$
$$V_x = V_{1x} + V_{2x}$$
$$V_y = V_{1y} + V_{2y}$$



Terna de ejes y versores:



$$\vec{\mathbf{V}} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}}$$

$$\begin{aligned}\vec{\mathbf{V}} = (V_x)\hat{\mathbf{i}} + (V_y)\hat{\mathbf{j}} &= \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 \\ &= (V_{1x}\hat{\mathbf{i}} + V_{1y}\hat{\mathbf{j}}) + (V_{2x}\hat{\mathbf{i}} + V_{2y}\hat{\mathbf{j}}) \\ &= (V_{1x} + V_{2x})\hat{\mathbf{i}} + (V_{1y} + V_{2y})\hat{\mathbf{j}}\end{aligned}$$

Cinemática vectorial:

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$$

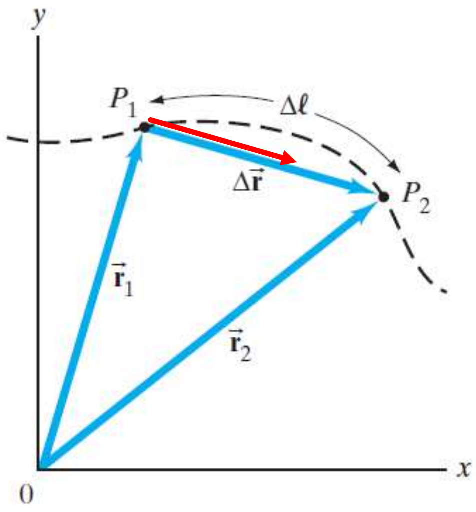
$$\Delta t = t_2 - t_1$$

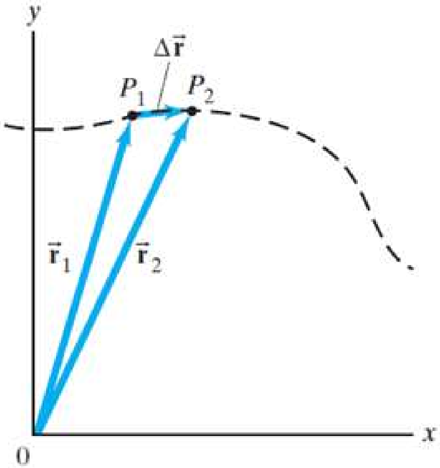
$$\vec{\mathbf{r}}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}},$$

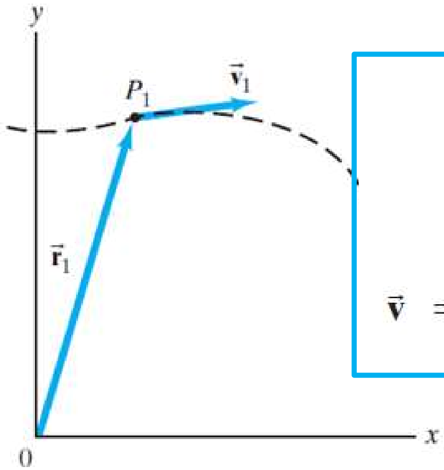
$$\vec{\mathbf{r}}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$$

$$\Delta\vec{\mathbf{r}} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}$$

$$\text{velocidad promedio} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



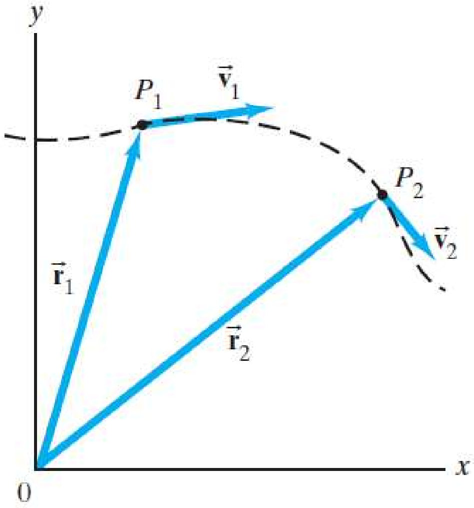




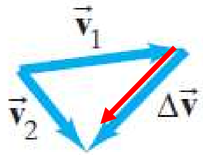
velocidad instantánea

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$



$$\boxed{\text{aceleración promedio} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1}}$$



$$\boxed{\vec{\mathbf{a}} = \lim_{\Delta\to 0}\frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt} \qquad \text{aceleración instantánea}}$$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}} + \frac{dv_z}{dt}\hat{\mathbf{k}}$$

$$= a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{a}} = \frac{d^2x}{dt^2}\hat{\mathbf{i}} + \frac{d^2y}{dt^2}\hat{\mathbf{j}} + \frac{d^2z}{dt^2}\hat{\mathbf{k}}$$

EJEMPLO **Posición dada como función del tiempo.** La posición de una partícula como una función del tiempo está dada por

$$\vec{\mathbf{r}} = \left[(5.0\,\text{m/s})t + (6.0\,\text{m/s}^2)t^2\right]\hat{\mathbf{i}} + \left[(7.0\,\text{m}) - (3.0\,\text{m/s}^3)t^3\right]\hat{\mathbf{j}},$$

donde r está en metros y t en segundos. *a*) ¿Cuál es el desplazamiento de la partícula entre $t_1 = 2.0$ s y $t_2 = 3.0$ s? *b*) Determine la velocidad instantánea y aceleración de la partícula como una función del tiempo. *c*) Evalúe $\vec{\mathbf{v}}$ y $\vec{\mathbf{a}}$ en $t = 3.0$ s.

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = (69\,\text{m} - 34\,\text{m})\hat{\mathbf{i}} + (-74\,\text{m} + 17\,\text{m})\hat{\mathbf{j}} = (35\,\text{m})\hat{\mathbf{i}} - (57\,\text{m})\hat{\mathbf{j}}.$$

$\Delta x = 35$ m y $\Delta y = -57$ m.

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \left[5.0\,\text{m/s} + (12\,\text{m/s}^2)t\right]\hat{\mathbf{i}} + \left[0 - (9.0\,\text{m/s}^3)t^2\right]\hat{\mathbf{j}}.$$

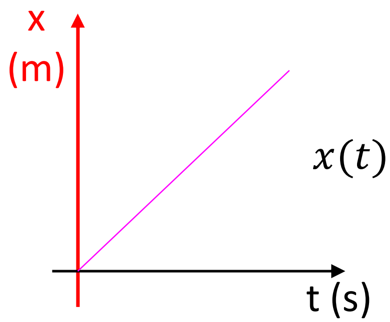
$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = (12\,\text{m/s}^2)\hat{\mathbf{i}} - (18\,\text{m/s}^3)t\,\hat{\mathbf{j}}.$$

Ecuaciones cinemáticas para aceleración constante en 2 dimensiones.	
Componente x (horizontal)	**Componente y (vertical)**
$v_x = v_{x0} + a_x t$	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

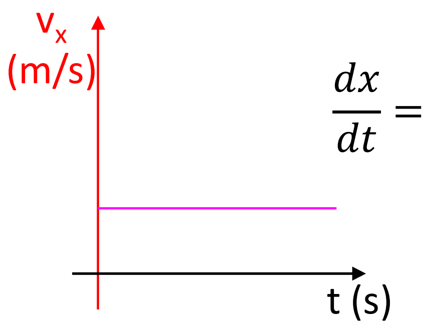
$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t \qquad [\vec{\mathbf{a}} = \text{constante}]$$

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 \qquad [\vec{\mathbf{a}} = \text{constante}]$$

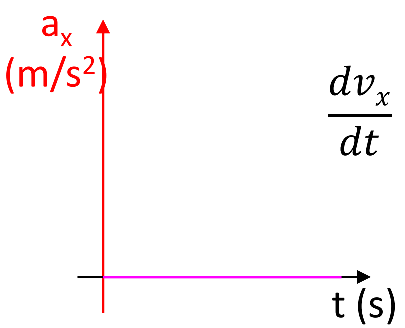
MRU



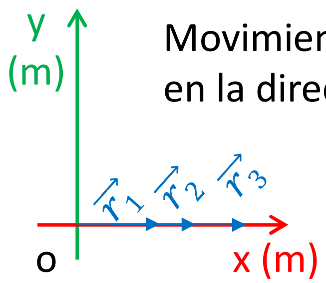
$$x(t) = x_o + v_{ox}t$$



$$\frac{dx}{dt} = v_x(t) = v_{ox} = cte$$



$$\frac{dv_x}{dt} = a_x(t) = 0$$



Movimiento rectilíneo
en la dirección x

$$\overrightarrow{r(t)} = (x(t)\hat{\imath} + y(t)\hat{\jmath})$$

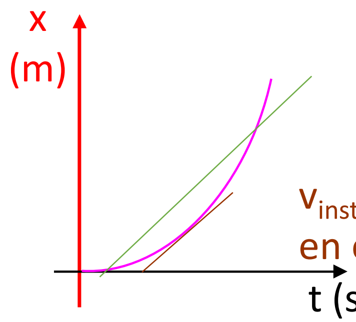
y(t) = 0 (porque no hay movimiento en y)

x(t) = depende de si es MRU o MRUV

$$\overrightarrow{v(t)} = \frac{d\overrightarrow{r(t)}}{dt} = (v_x(t)\hat{\imath} + v_y(t)\hat{\jmath})$$

$$\overrightarrow{a(t)} = \frac{d\overrightarrow{v(t)}}{dt} = (a_x(t)\hat{\imath} + a_y(t)\hat{\jmath})$$

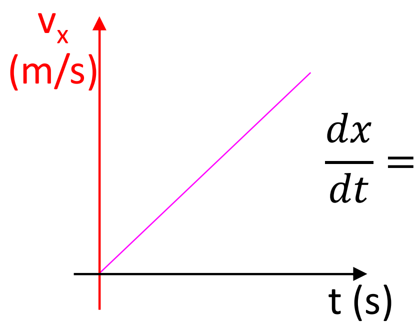
MRUV (supongo a_x>0)



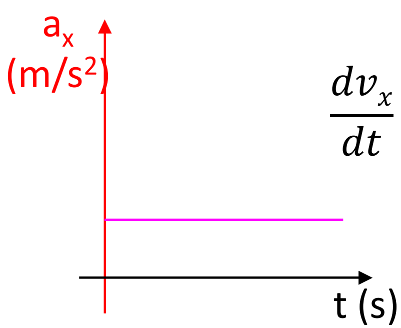
$$v_{media} = \frac{x_f - x_0}{t_f - t_0}$$

v_{inst}= pendiente recta tg en ese punto

$$x(t) = x_o + v_{ox}t + \frac{1}{2}a_x t^2$$

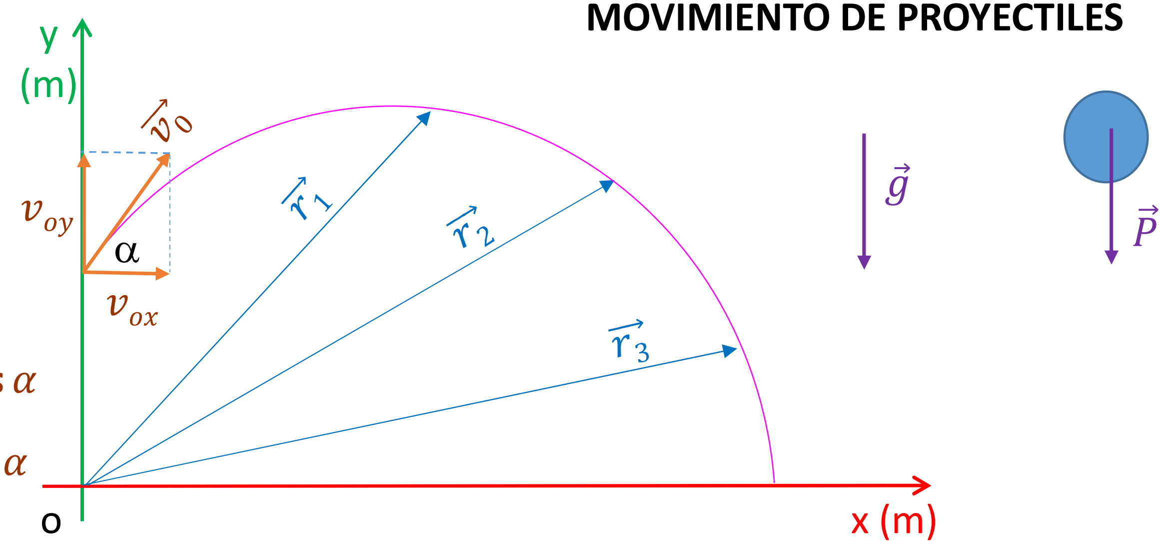


$$\frac{dx}{dt} = v_x(t) = v_{ox} + a_x t$$



$$\frac{dv_x}{dt} = a_x(t) = cte$$

MOVIMIENTO DE PROYECTILES



$$v_{ox} = v_0 \cos\alpha$$

$$v_{oy} = v_0 \sin\alpha$$

$$\overrightarrow{r(t)} = \ (x(t)\hat{\imath} + y(t)\hat{\jmath})$$

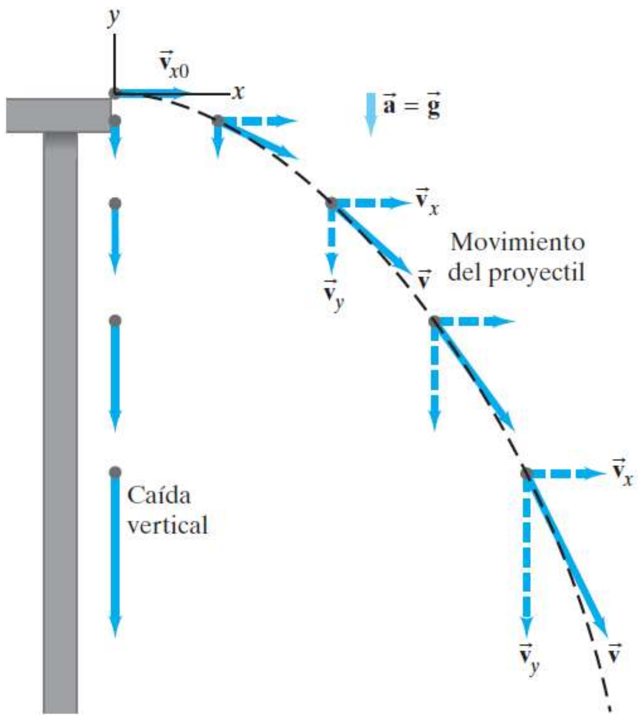
En x => MRU (no hay aceleración en x), v_x = cte = v_{ox}

$$x(t) = x_o + v_{ox}t$$

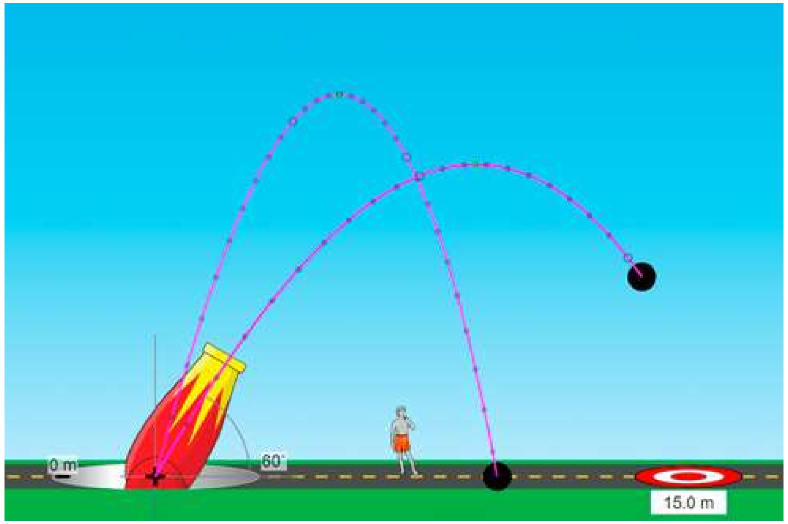
En y => MRUV; a_y = -9,8m/s^2

$$y(t) = y_o + v_{oy}t + \frac{1}{2}a_y t^2$$

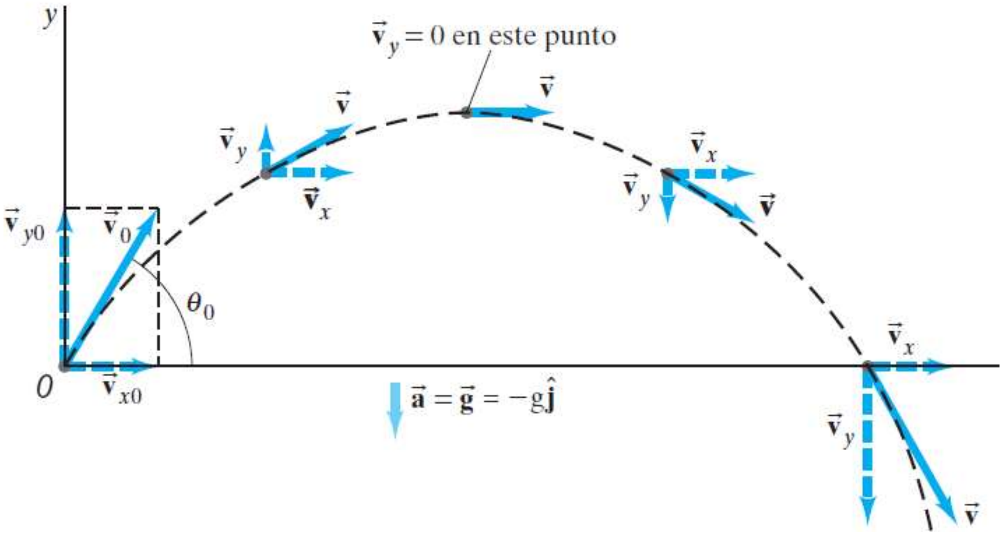
$$v_y(t) = v_{oy} + a_y t$$



y
$\vec{v}_{x0}$
x
$\vec{a} = \vec{g}$
$\vec{v}_x$
Movimiento
del proyectil
$\vec{v}_y$
$\vec{v}$
Caída
vertical

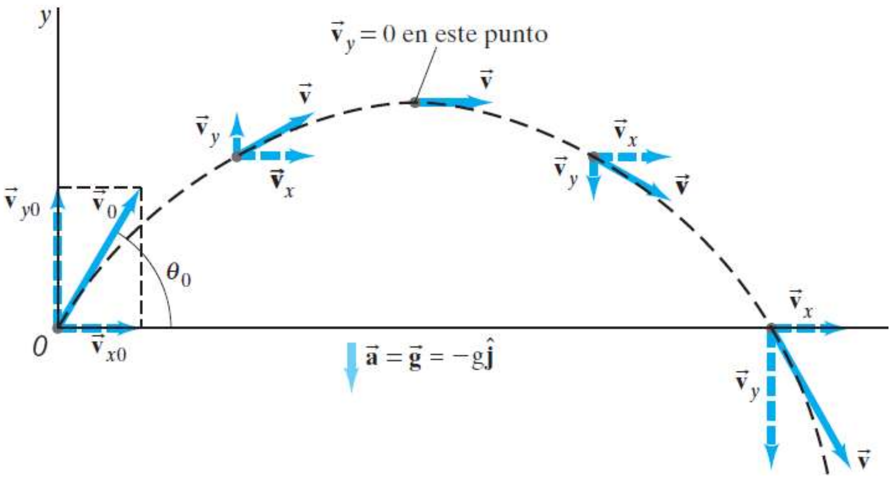


0 m
60°
15.0 m



y
$\vec{v}_y = 0$ en este punto
$\vec{v}$
$\vec{v}_y$
$\vec{v}_x$
$\vec{v}_{y0}$
$\vec{v}_0$
θ_0
0
$\vec{v}_{x0}$
$\vec{a} = \vec{g} = -g\hat{j}$

¿Cómo resolver?



$$v_{x0} = v_0 \cos\theta_0$$
$$v_{y0} = v_0 \operatorname{sen}\theta_0$$

Ecuaciones cinemáticas para el movimiento de proyectiles
(y positivo hacia arriba; $a_x = 0$, $a_y = -g = -9.80\ \mathrm{m/s^2}$)

Movimiento horizontal ($a_x = 0$, $v_x =$ constante)	Movimiento vertical ($a_y = -g =$ constante)
$v_x = v_{x0}$	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

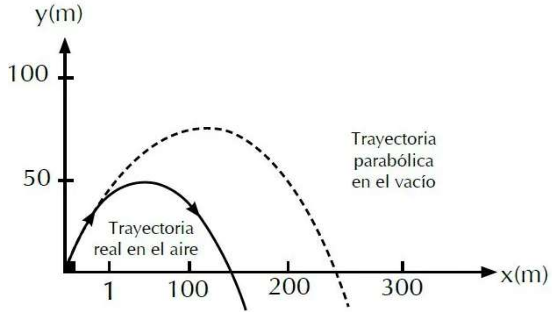
El movimiento de proyectiles es parabólico (en ausencia de fricción con el aire):

$$x = v_{x0}t$$

$$y = v_{y0}t - \tfrac{1}{2}gt^2$$

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2$$

$$y = Ax - Bx^2$$



Gráficas cinemáticas: